

7. Physics of the Higgs Boson

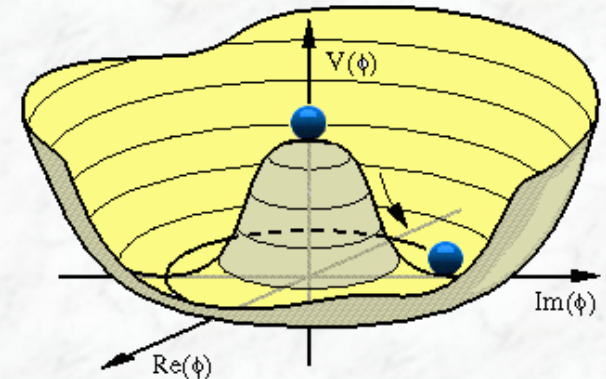
7.1 The Higgs boson in the Standard Model

7.2 Properties of the Higgs boson

7.3 Higgs boson production at hadron colliders

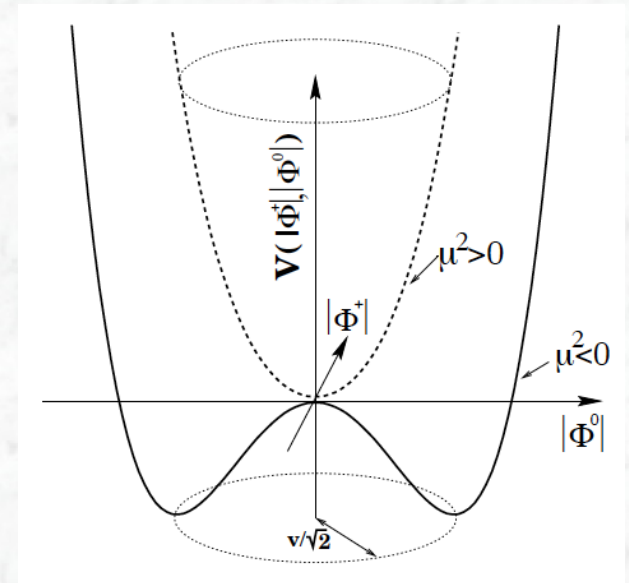
7.4 The search for and discovery of a Higgs boson at the LHC

7.5 What are its properties? Is it the Higgs boson of the Standard Model?



7.1 The Higgs boson in the Standard Model

(a brief summary, for more details, see lecture notes)



The structure of the Standard Model

Fundamental principle: **Local gauge invariance**
Prototype: **Quantum Electrodynamics (QED)**

Free Dirac equation: $i\gamma^\mu \partial_\mu \psi - m\psi = 0$

Lagrangian formalism: $L = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$

Local gauge transformation: $\psi(x) \rightarrow e^{i\alpha(x)}\psi(x)$

(derivative: $\partial_\mu \psi \rightarrow e^{i\alpha(x)}\partial_\mu \psi + ie^{i\alpha(x)}\psi\partial_\mu \alpha$,
 $\delta_\mu \alpha$ term breaks the invariance of L)

Invariance of L under **local gauge transformations** can be accomplished by introducing a **gauge field** A_μ , which transforms as:

$$A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu \alpha \quad \text{where } e = g_e/4\pi = \text{coupling strength}$$

Can be formally achieved by the construction of a “modified” derivative

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu \quad (\text{covariant derivative})$$

→ Lagrangian of QED:

$$L = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi + \underbrace{e\bar{\psi}\gamma^\mu A_\mu\psi}_{\text{interaction term}} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where $F_{\mu\nu}$ is the usual field strength tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Note:

- (i) Imposing local gauge invariance leads to the interacting field theory of QED
- (ii) **A mass term ($\frac{1}{2}m^2 A_\mu A^\mu$) for the gauge field A_μ would violate gauge invariance**

Similar for the Standard Model interactions:

Quantum Chromodynamics (QCD):

SU(3) transformations, 8 gauge fields,
8 massless gluons, Gluon self-coupling

- T_a ($a = 1, \dots, 8$) generators of the SU(3) group
(independent traceless 3x3 matrices)
- G_μ gluon fields
- g = coupling constant

$$D_\mu = \partial_\mu + igT_a G_\mu^a$$

$$G_\mu^a \rightarrow G_\mu^a - \frac{1}{g} \partial_\mu \alpha_a - f_{abc} \alpha_b G_\mu^c$$

Electroweak Interaction (Glashow, Salam, Weinberg):

SU(2)_L x U(1)_Y transformations,
4 gauge fields, ($W_\mu^1, W_\mu^2, W_\mu^3, B_\mu$)

Physical states:

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2)$$

$$Z_\mu = -\sin \theta_W B_\mu + \cos \theta_W W_\mu^3$$

$$A_\mu = \cos \theta_W B_\mu + \sin \theta_W W_\mu^3$$

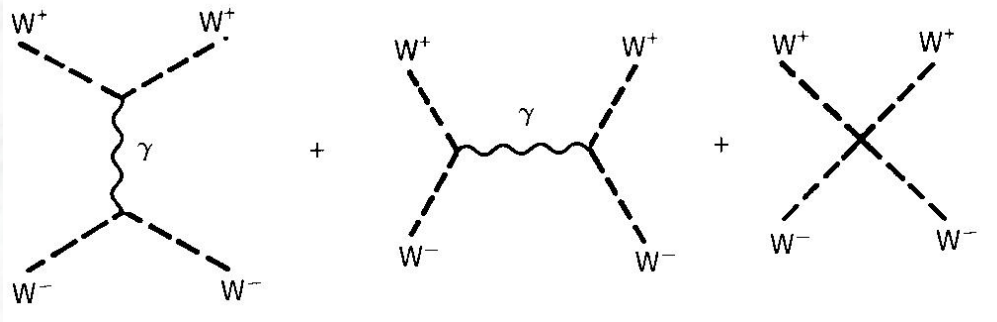
Problems at that stage:

- Masses of the vector bosons W and Z:**

Experimental results: $m_W = 80.385 \pm 0.015 \text{ GeV} / c^2$
 $m_Z = 91.1875 \pm 0.0021 \text{ GeV} / c^2$

A local gauge invariant theory requires massless gauge fields

- Divergences in the theory (scattering of W bosons)**



$$-iM(W^+W^- \rightarrow W^+W^-) \sim \frac{s}{M_W^2} \quad \text{for} \quad s \rightarrow \infty$$

Solution to **both** problems:

- create mass via spontaneous breaking of electroweak symmetry
- introduce a scalar particle that regulates the WW scattering amplitude

➔ **Higgs Mechanism**

The Higgs mechanism

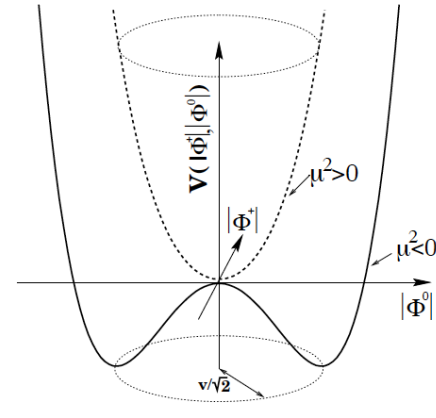
Spontaneous breaking of the SU(2) x U(1) gauge symmetry

- Scalar fields are introduced

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Potential :

$$V(\phi) = \mu^2 (\phi^* \phi) + \lambda (\phi^* \phi)^2$$



- Lagrangian for the scalar fields:
g, g' = SU(2), U(1) gauge couplings

$$L_2 = \left| \left(i\partial_\mu - g\mathbf{T} \cdot \mathbf{W}_\mu - g' \frac{Y}{2} B_\mu \right) \phi \right|^2 - V(\phi)$$

- For $\mu^2 < 0$, $\lambda > 0$,
minimum of potential:

$$\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = v^2 \quad v^2 = -\mu^2 / \lambda$$

- Perturbation theory around
ground state:

$$\phi_0(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \Rightarrow$$

Particle content and masses

- Mass terms for the W^\pm bosons:

$$m_{W^\pm} = \frac{1}{2} v g$$

- Remaining terms off-diagonal in W_μ^3 and B_μ :

$$\frac{1}{8} v^2 (W_\mu^3, B_\mu) \begin{pmatrix} g^2 & -g g' \\ -g g' & g'^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} = \frac{1}{8} v^2 \left[g W_\mu^3 - g' B_\mu \right]^2 + 0 \left[g' W_\mu^3 + g B_\mu \right]^2$$

- Massless photon:

$$A_\mu = \frac{g' W_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}} \quad \text{with} \quad m_A = 0$$

- Massive neutral vector boson: $Z_\mu = \frac{g W_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$ with

$$m_Z = \frac{1}{2} v \sqrt{g^2 + g'^2}$$

Masses of the gauge bosons:

$$\begin{aligned}
 & \left| \left(-ig \frac{\boldsymbol{\tau}}{2} \cdot \mathbf{W}_\mu - i \frac{g'}{2} B \right) \phi \right|^2 \\
 &= \frac{1}{8} \left| \begin{pmatrix} gW_\mu^3 + g'B_\mu & g(W_\mu^1 - iW_\mu^2) \\ g(W_\mu^1 + iW_\mu^2) & -gW_\mu^3 + g'B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\
 &= \frac{1}{8} v^2 g^2 \left[(W_\mu^1)^2 + (W_\mu^2)^2 \right] + \frac{1}{8} v^2 (g'B_\mu - gW_\mu^3)(g'B^\mu - gW^{3\mu}) \\
 &= \left(\frac{1}{2} v g \right)^2 W_\mu^+ W^{-\mu} + \frac{1}{8} v^2 (W_\mu^3, B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}
 \end{aligned}$$

Important relations in the Glashow-Salam-Weinberg model:

- Relation between the gauge couplings:

$$\frac{g'}{g} = \tan \theta_w$$

→ Important prediction of the GSW with a Higgs doublet:

$$\frac{m_W}{m_Z} = \cos \theta_w$$

or expressed in terms of the ρ parameter:

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_w} = 1$$

- From the M_W relation the value of the vacuum expectation value of the Higgs field can be calculated:

$$\frac{1}{2v^2} = \frac{g^2}{8m_W^2} = \frac{G_F}{\sqrt{2}} \quad \rightarrow \quad v = 246 \text{ GeV}$$

where G_F = Fermi constant, known from low energy experiments (muon decay)

Masses of the Fermions:

- The same Higgs doublet which generates W^\pm and Z masses is sufficient to give masses to the fermions (leptons and quarks):
e.g. for electrons: use an arbitrary coupling G_e

$$L_3 = -G_e \left[(\bar{\nu}_e, \bar{e})_L \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} e_R + \bar{e}_R (\phi^-, \bar{\phi}^0) \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right]$$

- Spontaneous symmetry breaking:

$$\phi = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

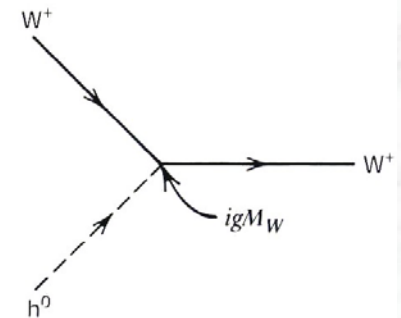
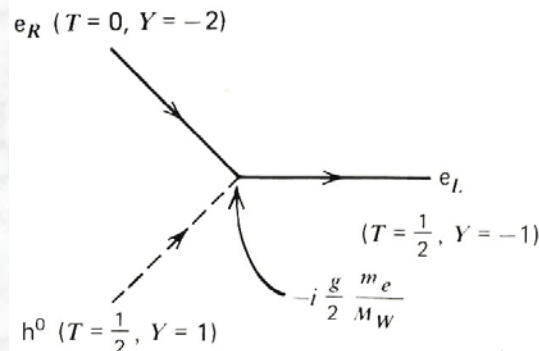
$$L_3 = -\frac{G_e v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) - \frac{G_e}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) h$$

mass term

interaction term with
the Higgs field

- Important relation: coupling of the Higgs boson to fermions is proportional to their mass

$$G_f = \frac{\sqrt{2} m_f}{v}$$



and finally..... a massive scalar with self-coupling, the **Higgs boson**:

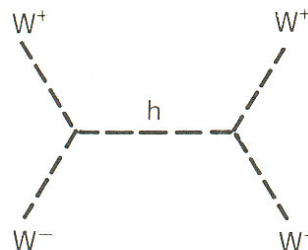
- Mass: $m_h^2 = 2v^2 \lambda$

(since λ is not predicted by theory, the mass of the Higgs boson is unknown)

- Self-coupling: $-\lambda v h^3 - \frac{1}{4} \lambda h^4$

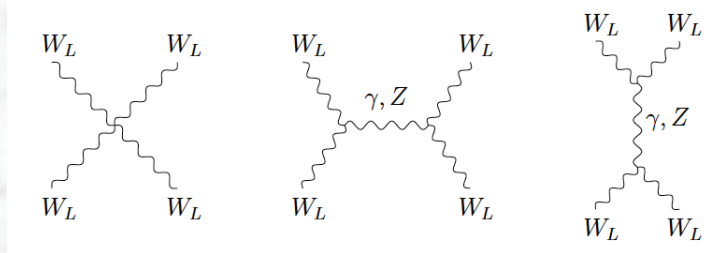
..... and:

- The additional diagram, with Higgs boson exchange, regulates the divergences in the longitudinal WW scattering



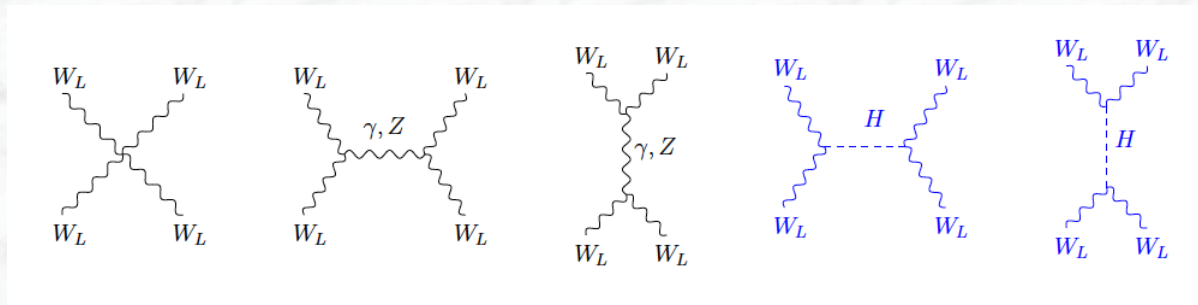
The Higgs boson as a UV regulator

Scattering of longitudinally polarized W bosons



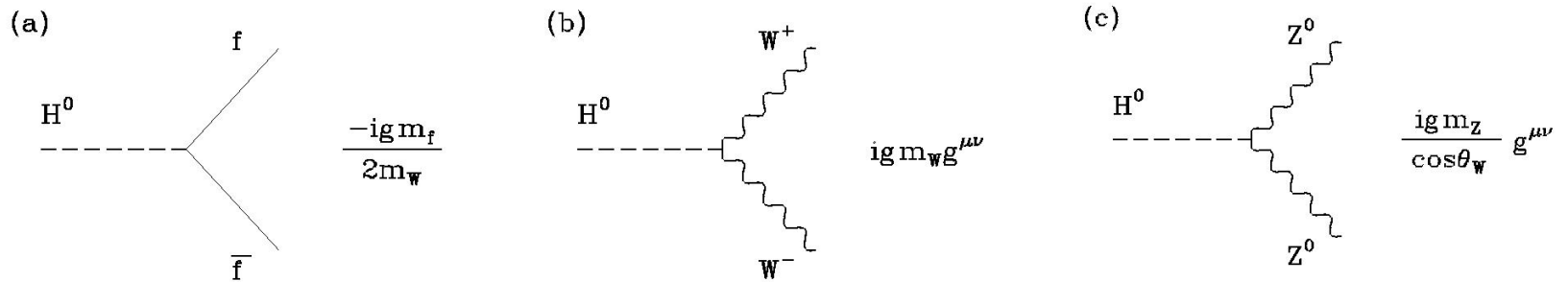
$$-iM(W^+W^- \rightarrow W^+W^-) \sim \frac{s}{m_W^2} \quad \text{for} \quad s \rightarrow \infty$$

Higgs boson guarantees unitarity (if its mass is $< \sim 1$ TeV)



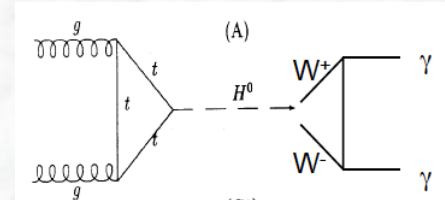
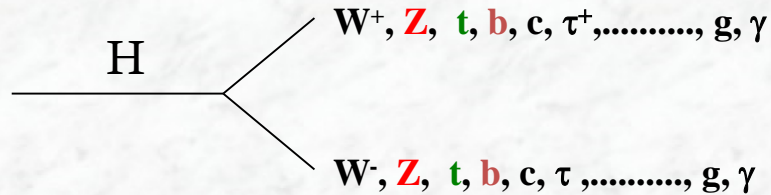
$$-iM(W^+W^- \rightarrow W^+W^-) \sim m_H^2 \quad \text{for} \quad s \rightarrow \infty$$

7.2 Higgs boson properties



Higgs Boson Decays

The decay properties of the Higgs boson are fixed, **if the mass is known**:



$$\Gamma(H \rightarrow \bar{f}f) = N_c \frac{G_F}{4\sqrt{2}\pi} m_f^2(m_H^2) m_H$$

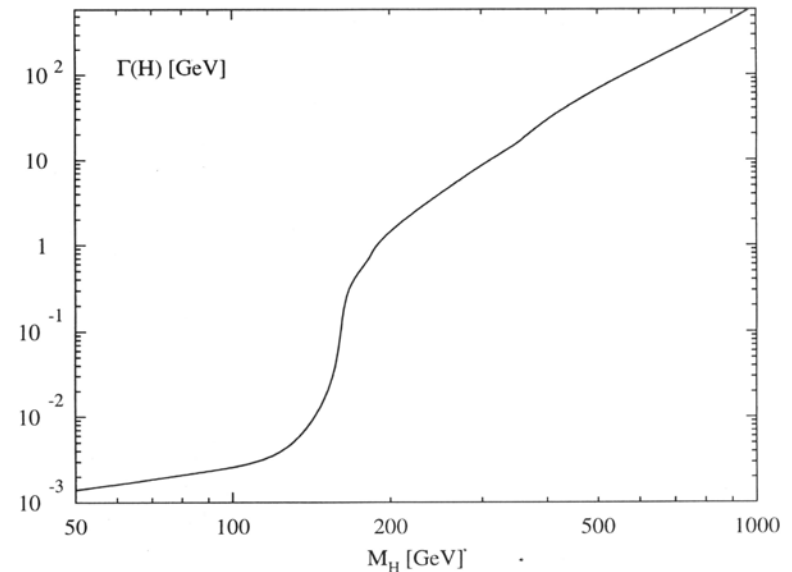
$$\Gamma(H \rightarrow WW) = \delta_V \frac{G_F}{16\sqrt{2}\pi} m_H^3 (1 - 4x + 12x^2) \beta_V$$

where: $\delta_Z = 1$, $\delta_W = 2$, $x = m_V^2 / m_H^2$, $\beta = \text{velocity}$

$$\Gamma(H \rightarrow gg) = \frac{G_F \alpha_a^2(m_H^2)}{36\sqrt{2}\pi^3} m_H^3 \left[1 + \left(\frac{95}{4} - \frac{7N_f}{6} \right) \frac{\alpha_a}{\pi} \right] \quad (+ W\text{-loop contributions})$$

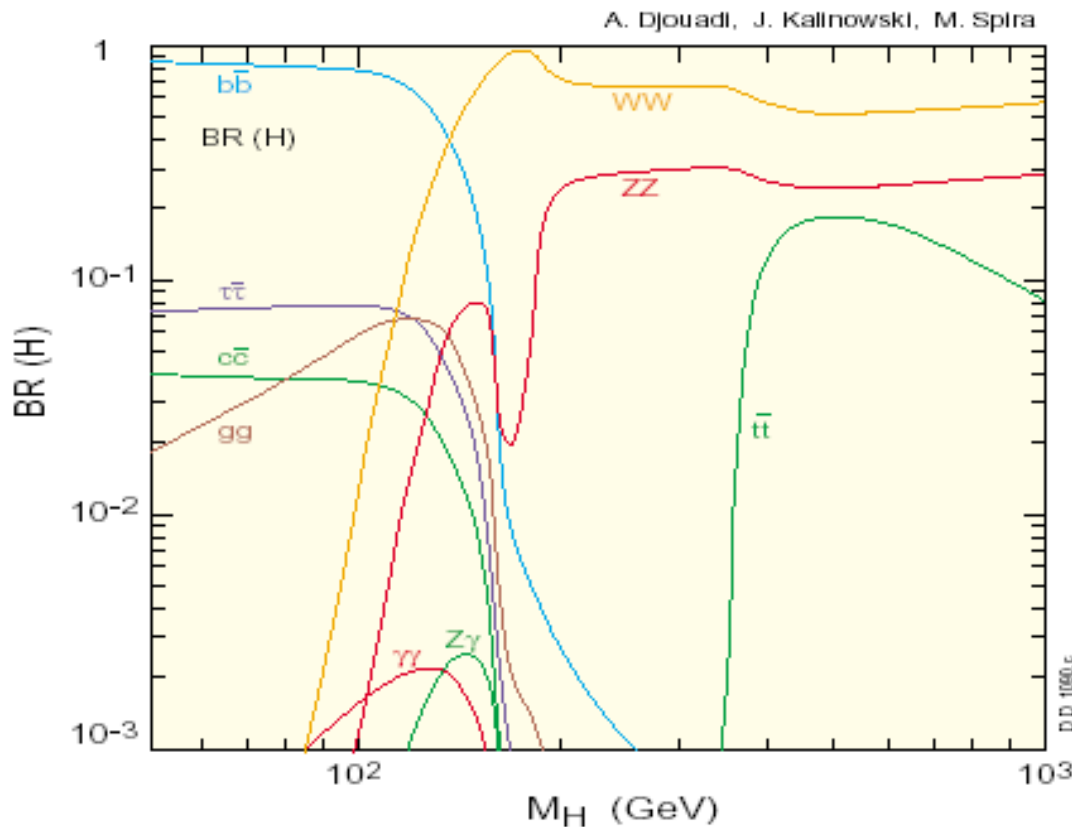
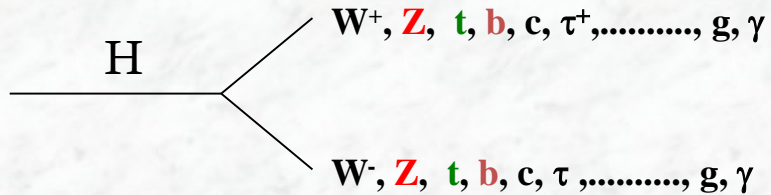
$$\Gamma(H \rightarrow \gamma\gamma) = \frac{G_F \alpha_a^2}{128\sqrt{2}\pi^3} m_H^3 \left[\frac{4}{3} N_c e_t^2 - 7 \right]^2$$

Total width



Higgs Boson Decays

The decay properties of the Higgs boson are fixed, **if the mass is known**:



Constraints on the Higgs boson mass

1. Constraints from theory
2. Indirect limits from electroweak precision data (theory and experiment)
3. Limits from Direct Searches (LEP, Tevatron)

(i) Theory Constraints on the Higgs boson mass

- Unitarity limit:

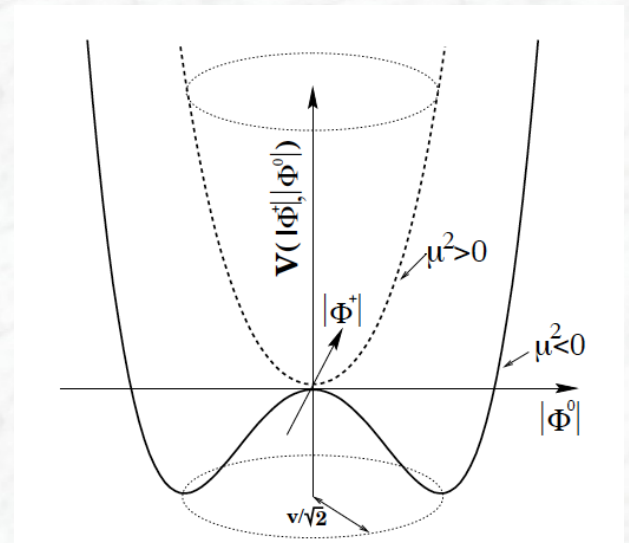
If Higgs boson too heavy, the regulation of the WW cross section is less effective and unitarity is violated again

→ $m_H < \sim 1 \text{ TeV}$ (as just discussed)

- Stricter limits from the energy dependence of the Higgs boson self coupling λ

- Stability of the vacuum
- Diverging coupling $\lambda(Q^2)$

→ next slides

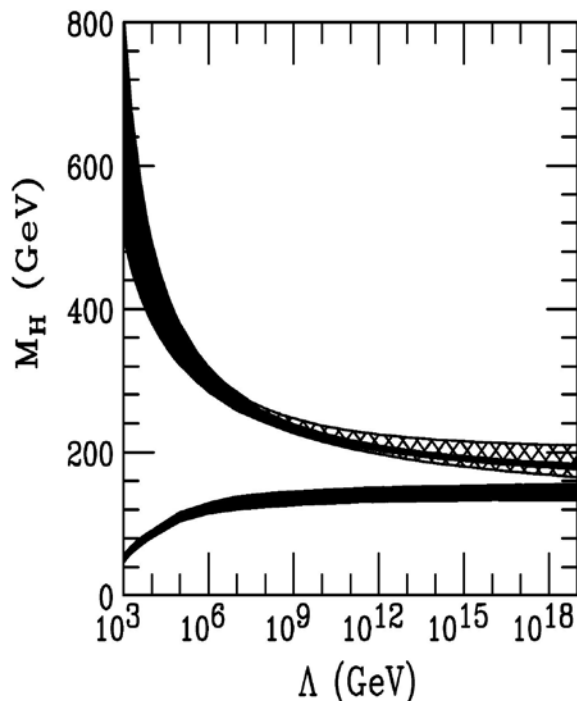


Tighter Higgs mass constraints:

Stronger bounds on the Higgs-boson mass result from the energy dependence of the Higgs coupling $\lambda(Q^2)$

(if the Standard Model is assumed to be valid up to some scale Λ)

$$\lambda(Q^2) = \lambda_0 \left\{ 1 + \frac{3\lambda_0}{2\pi^2} \log \left(2 \frac{Q^2}{v^2} \right) + \dots - \frac{3g_t^4}{32\pi^2} \log \left(2 \frac{Q^2}{v^2} \right) + \dots \right\} \quad \text{where} \quad \lambda_0 = \frac{m_h^2}{v^2}$$



Upper bound: diverging coupling
(Landau Pole)

Lower bound: stability of the vacuum
(negative contribution from
top quark dominates)

Mass bounds depend on scale Λ
up to which the Standard Model should be
valid

(ii) Indirect limits from electroweak precision data (m_W and m_t)

Motivation:

W mass and top quark mass are **fundamental parameters** of the Standard Model;
The standard theory provides well defined **relations between m_W , m_t and m_H**

Electromagnetic constant
measured in atomic transitions,
 e^+e^- machines, etc.

$$m_W = \left(\frac{\pi \alpha_{EM}}{\sqrt{2} G_F} \right)^{1/2} \frac{1}{\sin \theta_W \sqrt{1 - \Delta r}}$$

Fermi constant
measured in muon
decay

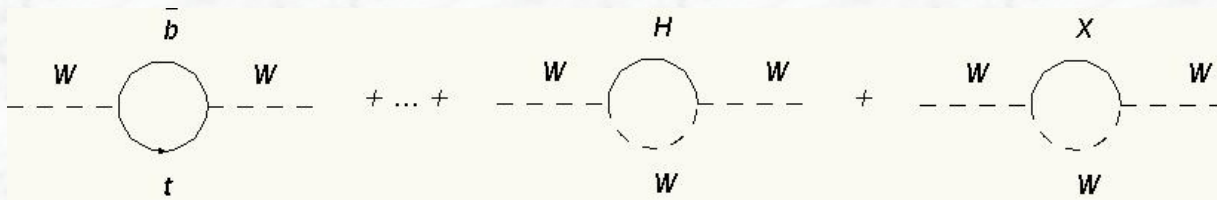
weak mixing angle
measured at
LEP/SLC

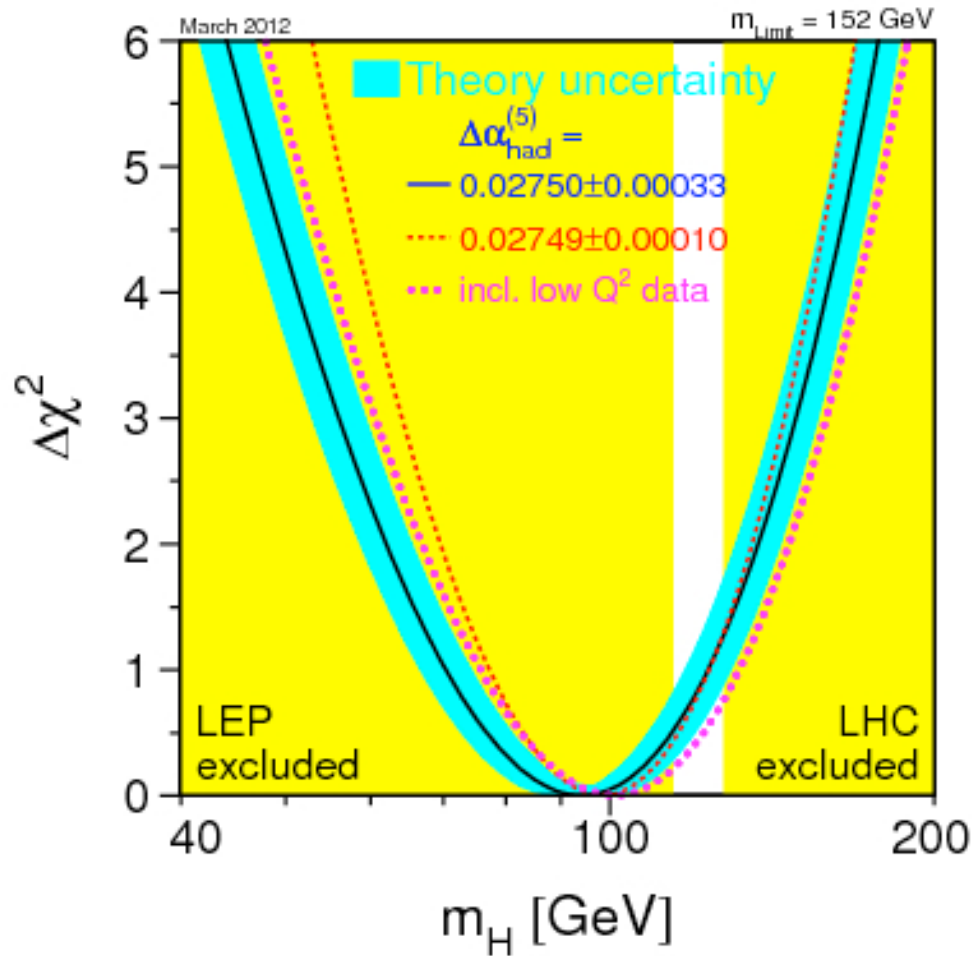
radiative corrections
 $\Delta r \sim f(m_t^2, \log m_H)$
 $\Delta r \approx 3\%$

G_F , α_{EM} , $\sin \theta_W$

are known with high precision

Precise measurements of the
W mass and the top-quark
mass constrain the Higgs-
boson mass
(and/or the theory,
radiative corrections)





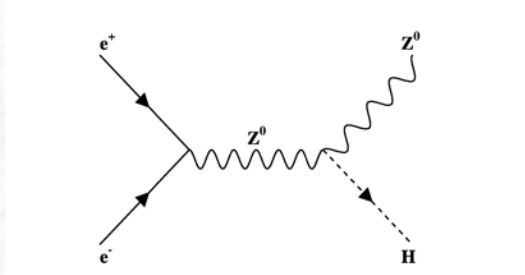
Results of the precision el.weak
measurements: (LEWWG-2012):

$$m_H = 94^{+29}_{-24} \text{ GeV}$$

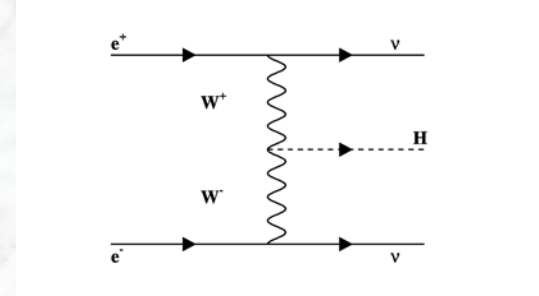
(iii) Constraints from
direct searches at LEP

Higgs bosons searches at LEP

Higgs-Strahlung: $e^+ e^- \rightarrow Z H$



WW-Fusion: $e^+ e^- \rightarrow \nu \nu H$



Higgs decay branching ratios for $m_H=115 \text{ GeV}/c^2$:

$\text{BR}(H \rightarrow b\bar{b}) = 74\%$, $\text{BR}(H \rightarrow \tau\tau, WW, gg) = 7\% \text{ each}$, $\text{BR}(H \rightarrow c\bar{c}) = 4\%$

Decay modes searched for:

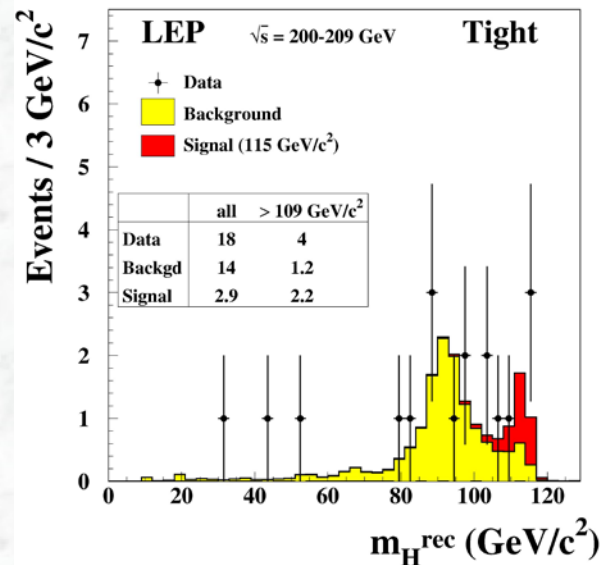
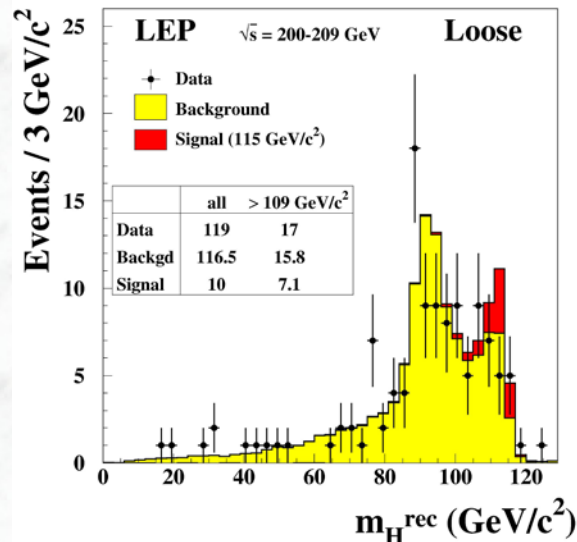
- Four Jet channel: $HZ \rightarrow b\bar{b} q\bar{q}$
- Missing energy channel: $\rightarrow b\bar{b} \nu\bar{\nu}$
- Leptonic channel: $\rightarrow b\bar{b} e\bar{e}, b\bar{b} \mu\bar{\mu}$
- Tau channels: $\rightarrow b\bar{b} \tau\bar{\tau}, \text{ and } \tau\bar{\tau} q\bar{q}$

Results of the final LEP analysis:

Final results have been published: [CERN-EP / 2003-011](#):

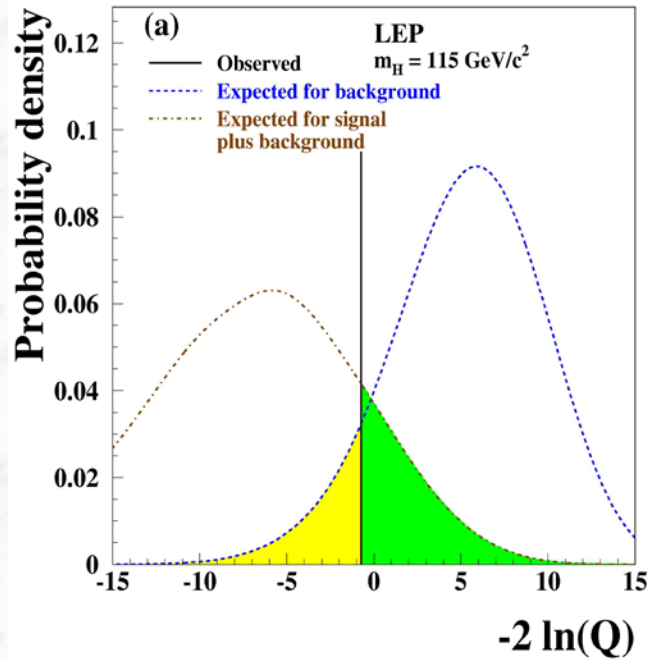
Based on final calibrations of the detectors, LEP-beam energies, final Monte Carlo simulations and analysis procedures.

The reconstructed $b\bar{b}$ mass for two levels of signal purity (loose and tight cuts):



Clear peak in the background prediction in the vicinity of m_Z due to the $e^+e^- \rightarrow ZZ$ background, which is consistent with the data.

Final combined LEP result



	$1 - \text{CL}_b$	CL_{s+b}
LEP	0.09	0.15
ALEPH	3.3×10^{-3}	0.87
DELPHI	0.79	0.03
L3	0.33	0.30
OPAL	0.50	0.14
Four-jet	0.05	0.44
All but four-jet	0.37	0.10

$$1 - \text{CL}_B = 0.09 \quad \leftrightarrow$$

Signal significance = 1.7σ

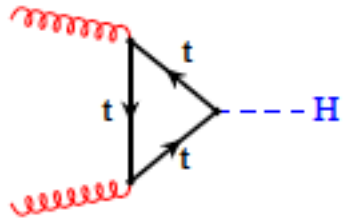
Likelihood ratio $Q := L_{S+B} / L_B$
 Test statistics: $-2 \ln Q$

$M_H > 114.4 \text{ GeV}/c^2$ (95% CL)

expected mass limit: $115.3 \text{ GeV}/c^2$
 (sensitivity)

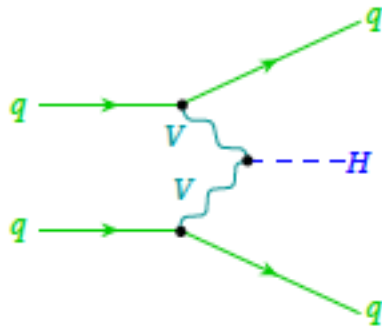
7.3 Higgs boson production at Hadron Colliders

Higgs Boson production processes at Hadron Colliders

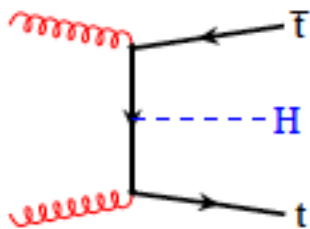


Gluon Fusion

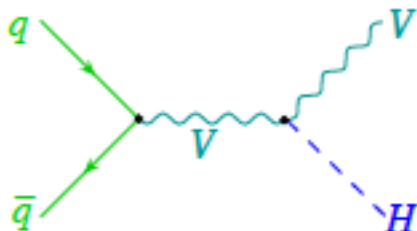
Relative importance of the various processes is different at the LHC and at the Tevatron



Vector boson fusion

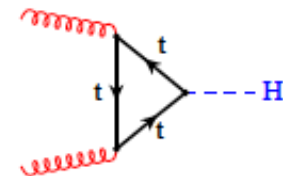
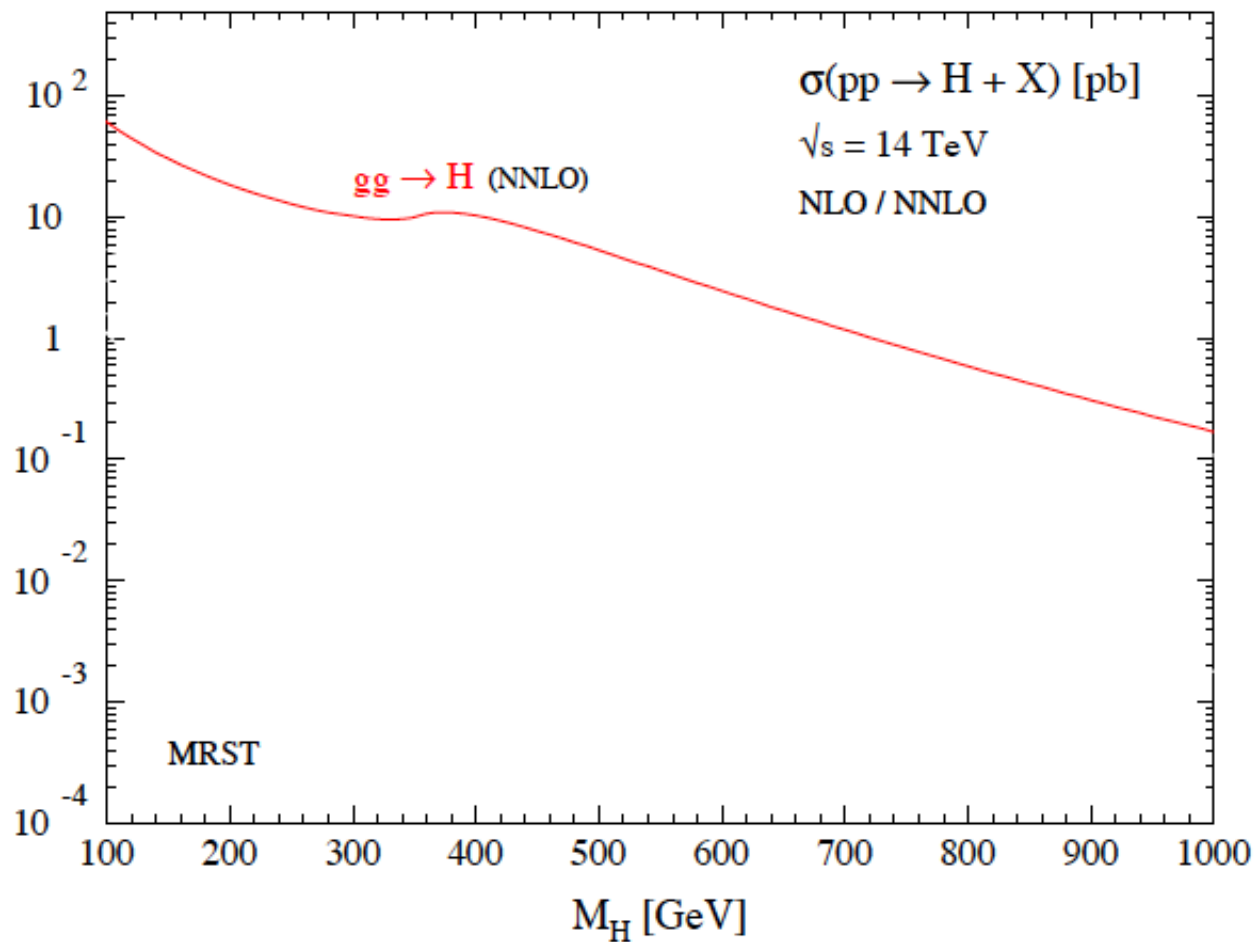


$t\bar{t}$ associated production



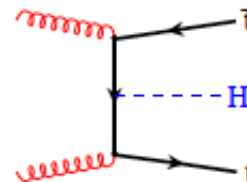
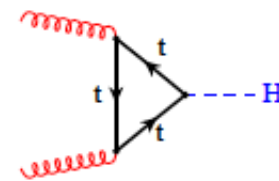
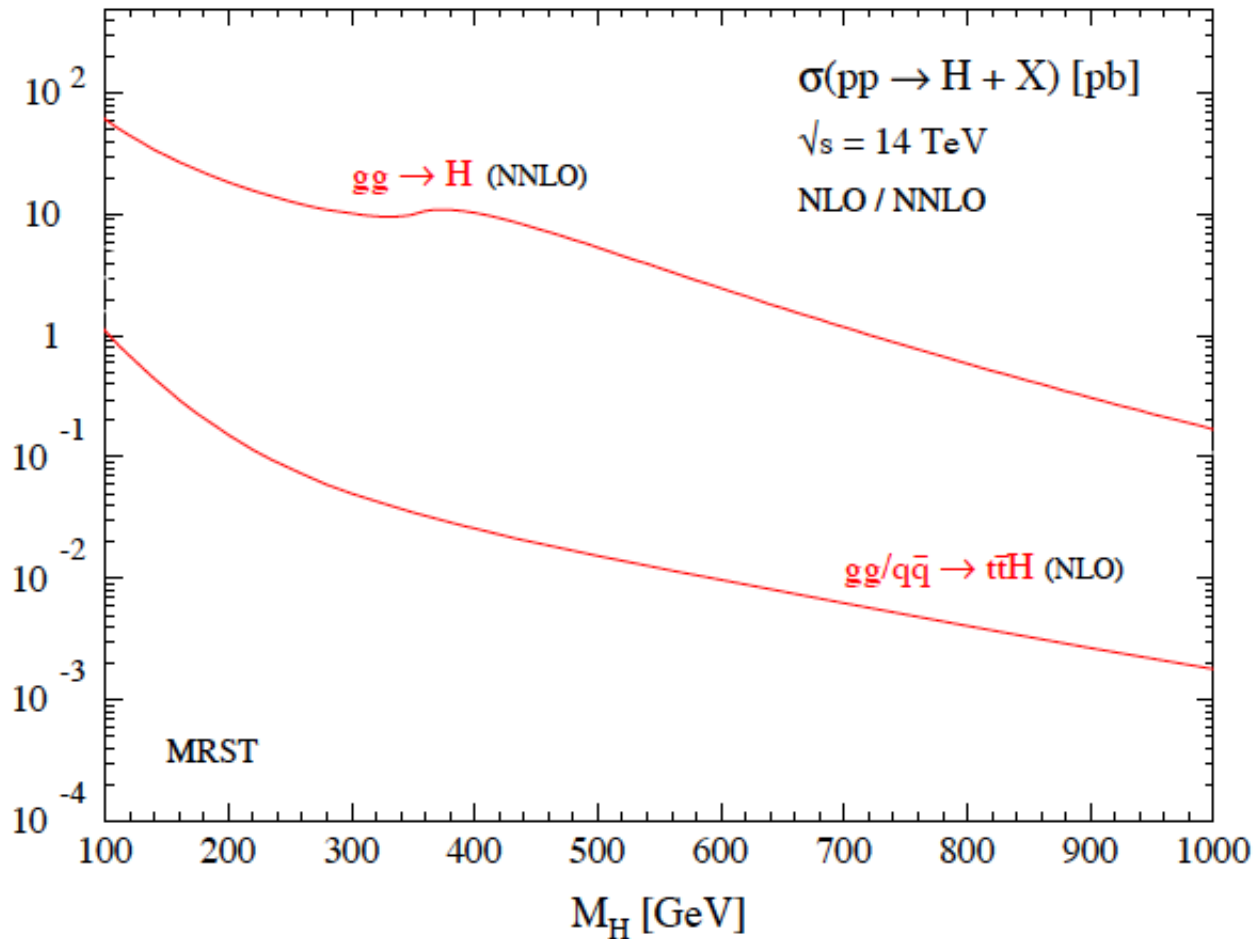
WH/ZH associated production

Production cross sections at the LHC



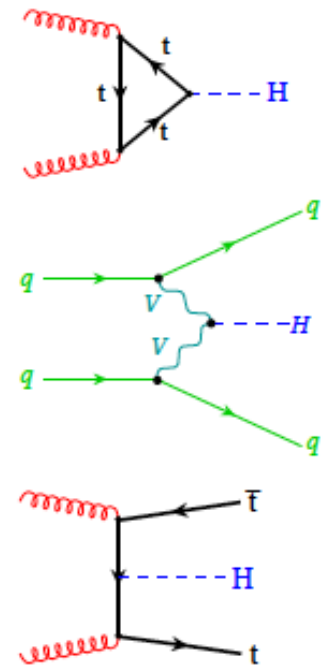
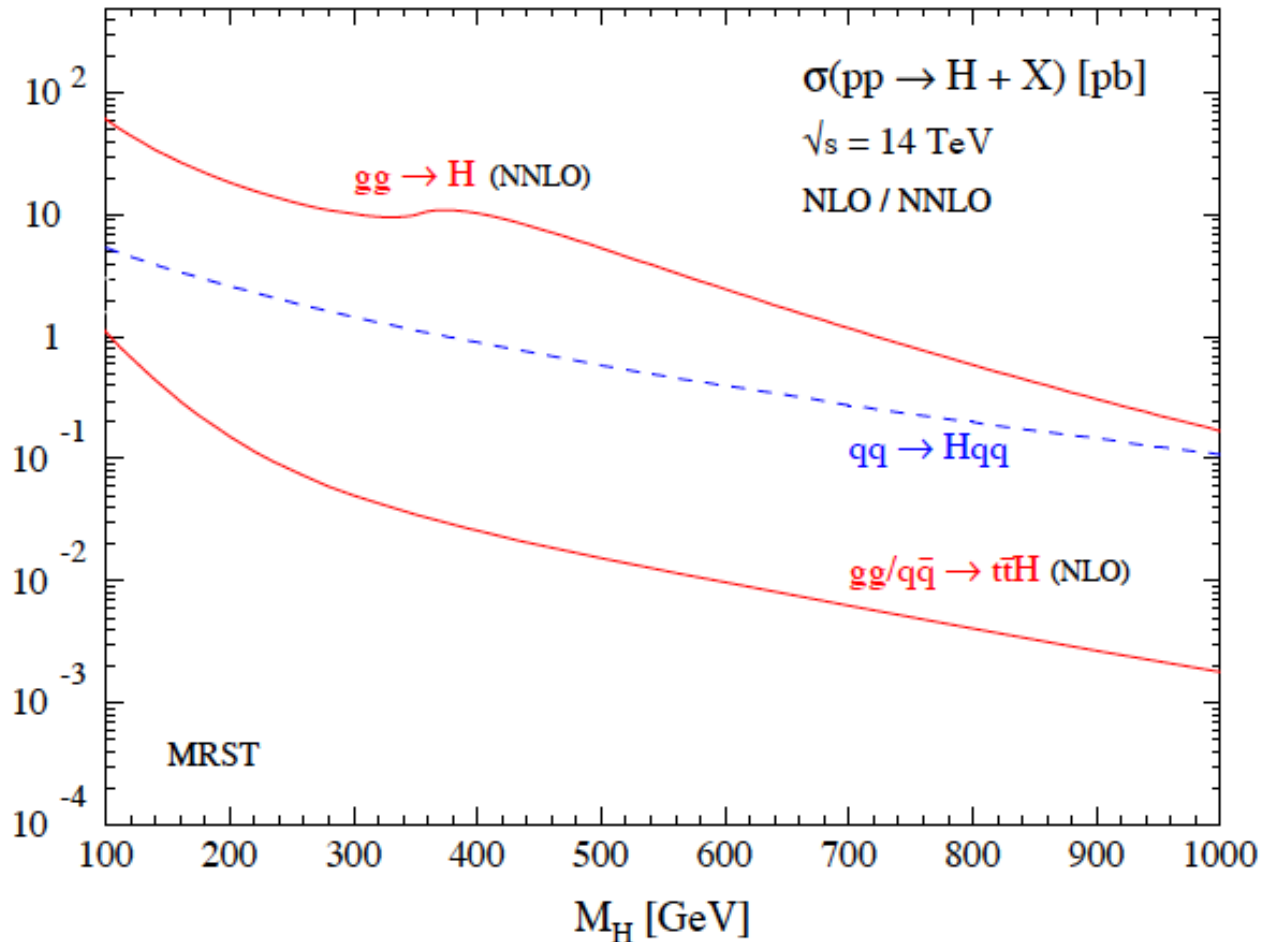
(for $\sqrt{s} = 14$ TeV, difference between 14 and 7 TeV to be discussed tomorrow)

Production cross sections at the LHC



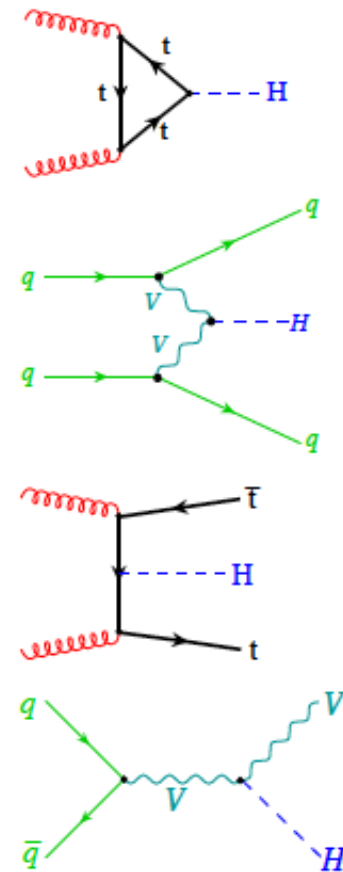
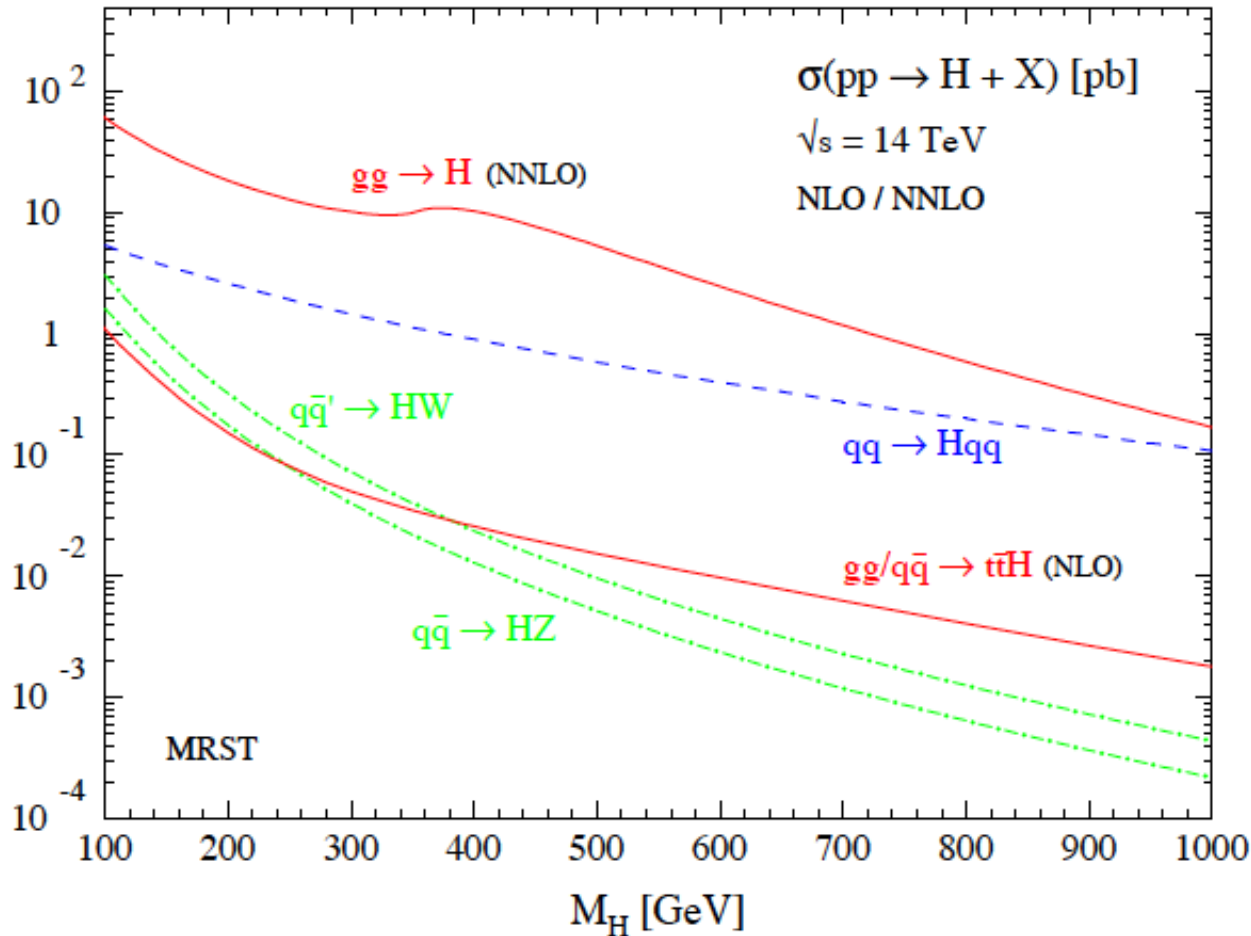
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Production cross sections at the LHC



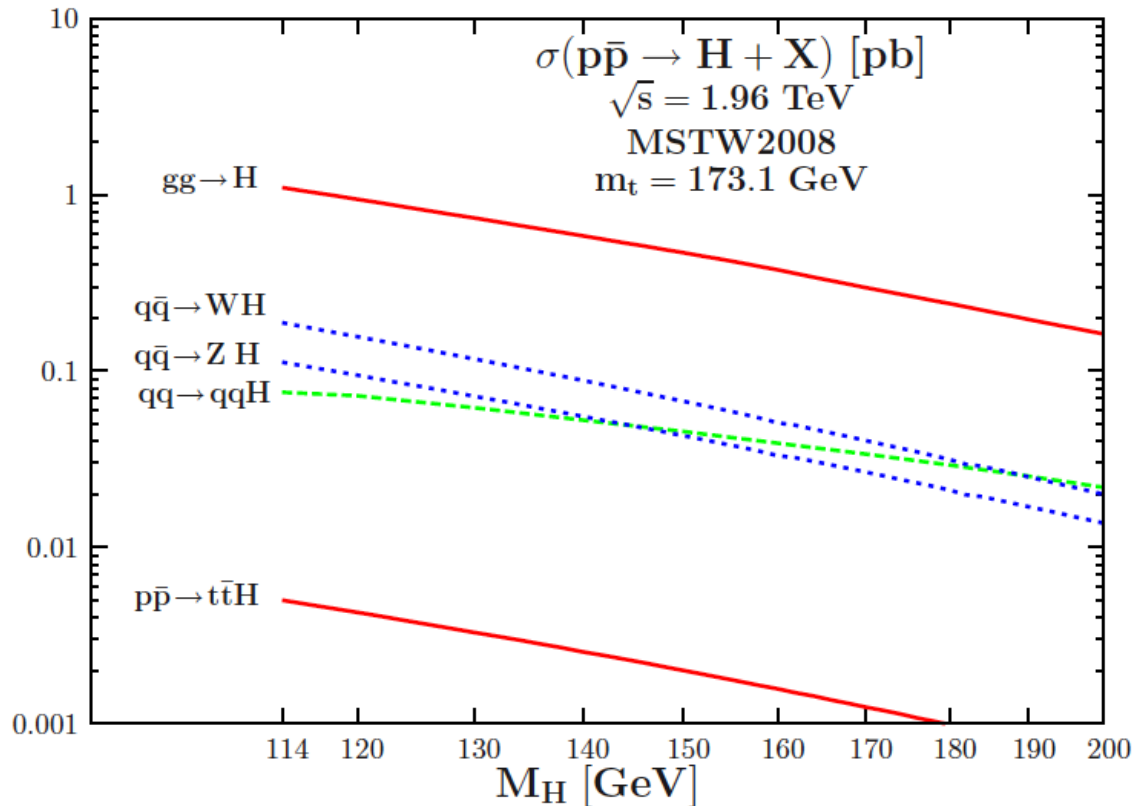
(for $\sqrt{s} = 14 \text{ TeV}$, difference between 14 and 7 TeV to be discussed tomorrow)

Production cross sections at the LHC



(for $\sqrt{s} = 14 \text{ TeV}$, difference between 14 and 7 TeV to be discussed tomorrow)

Production cross sections at the Tevatron



1. Gluon fusion

2./3. W/Z H associated production
Vector boson fusion

4. $t\bar{t}H$ (very small cross section)

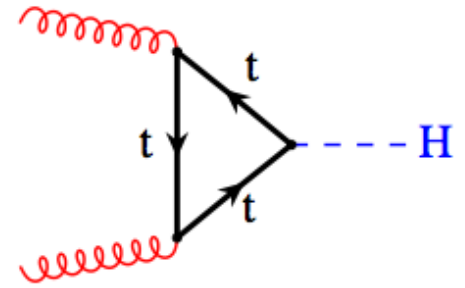
J. Baglio, A. Djouadi, arXiv:1003.4266

$qq \rightarrow W/Z + H$ cross sections
 $gg \rightarrow H$

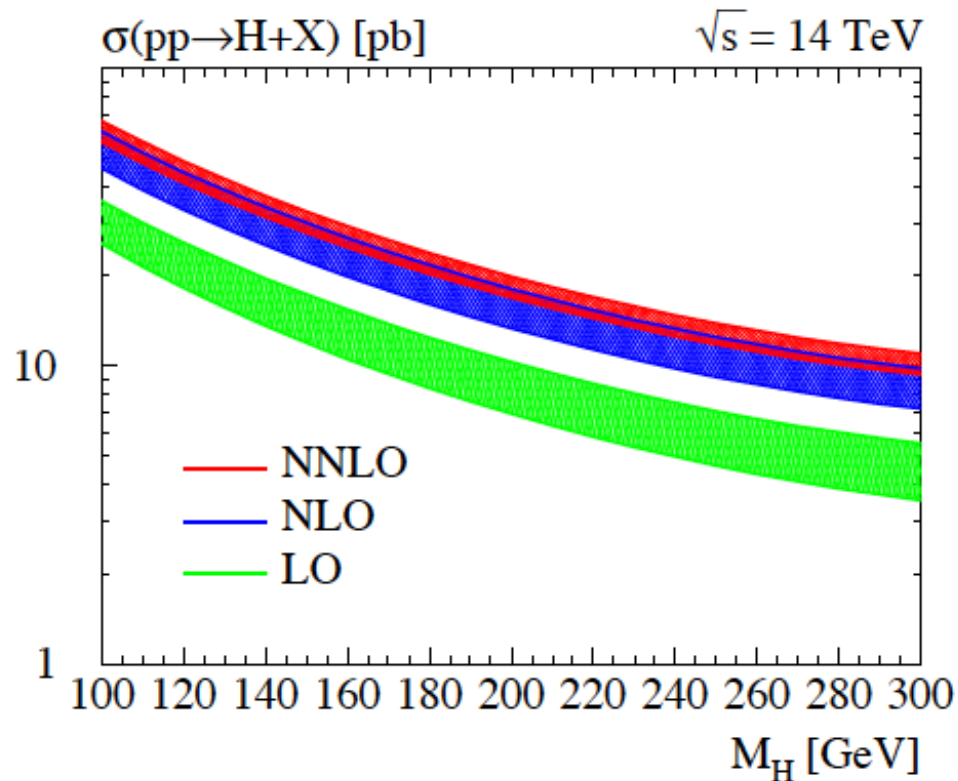
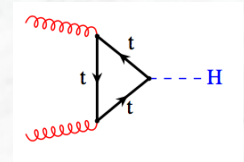
~10 x larger at the LHC ($\sqrt{s} = 14$ TeV)
 ~70-80 x larger at the LHC ($\sqrt{s} = 14$ TeV)

Gluon fusion:

- Dominant production mode
- Sensitive to heavy particle spectrum ...
(e.g. 4th generation quarks)
...and the corresponding Yukawa couplings
(important for coupling measurements, top Yukawa coupling)
- Large K-factors (NLO, NNLO corrections)
 - Difficult to calculate, loop already at leading order
(calculation with infinite top mass is used as an approximation, however, this seems to be a good approximation)
 - Nicely converging perturbative series



Higher order corrections:



- Spira, Djouadi, Graudenz, Zerwas (1991)
- Dawson (1991)

- Harlander, Kilgore (2002)
- Anastasiou, Melnikov (2002)
- Ravindran, Smith, van Neerven (2003)

Independent variation of renormalization and factorization scales
(with $0.5 m_H < \mu_F, \mu_R < 2 m_H$)

Vector boson fusion:

- Second largest production mode, Distinctive signature (forward jets, little jet activity in the central region)
- Sensitivity to W/Z couplings
- Moderate K-factors (NLO corrections)

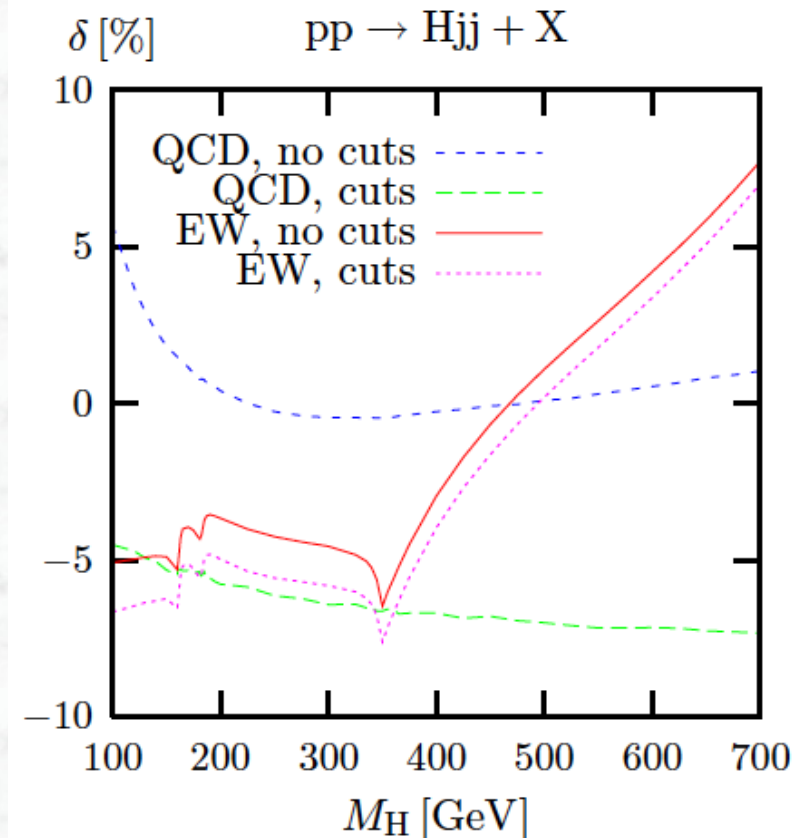
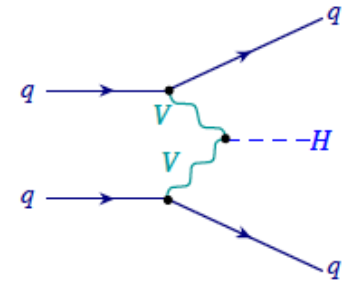
Both NLO QCD and el.weak have been calculated

- Effective K-factor depends on experimental cuts

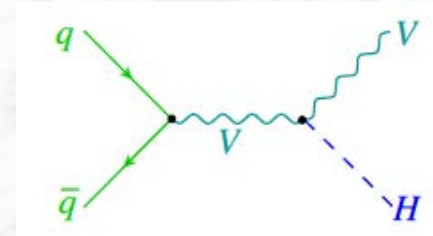
Example: typical VBF cuts

$P_T(\text{jet}) > 20 \text{ GeV}$

$\eta < 4.5, \Delta\eta > 4, \eta_1 \cdot \eta_2 < 0$



WH / ZH associated production:



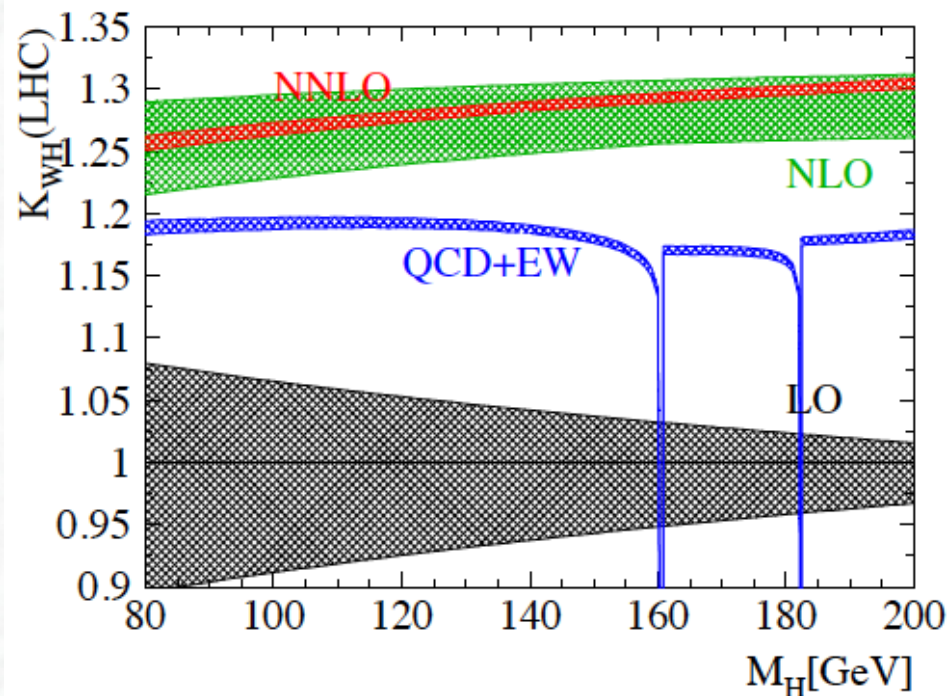
- Weak at the LHC,
Relatively stronger at the Tevatron
- Allows for a Higgs-decay-independent trigger
 $W \rightarrow l\nu$, $Z \rightarrow ll$
- Sensitivity to W/Z couplings
- Moderate K-factors
(NLO corrections)

Both NLO QCD and el.weak
corrections available

Brein, Djouadi, Harlander, (2003)

Han, Willenbrock (1990)

Ciccolini, Dittmaier, Krämer (2003)



ttH associated production:

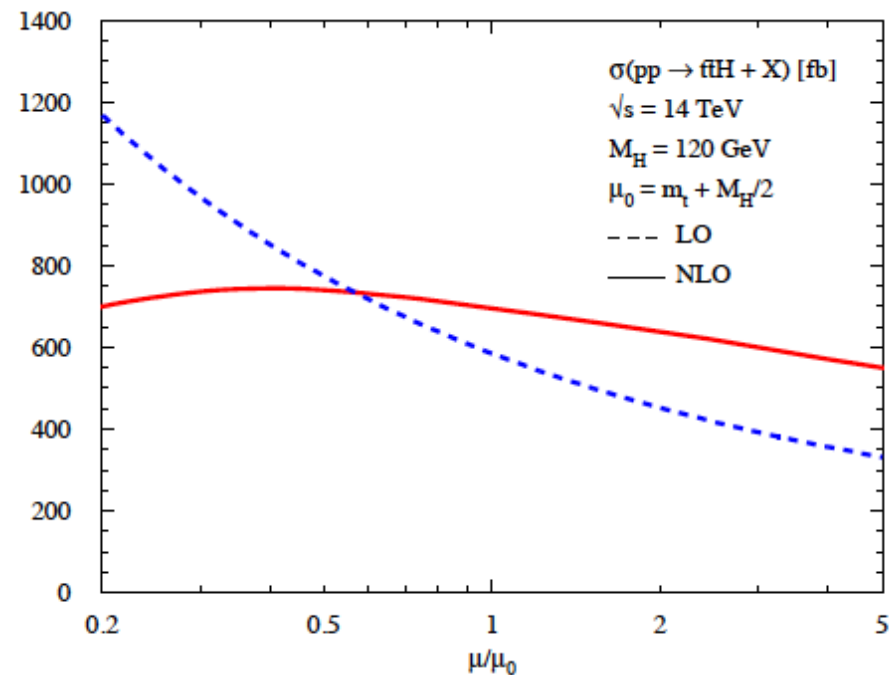
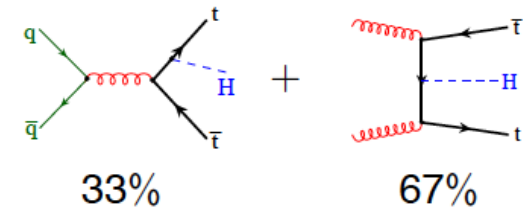
- Weak and difficult at the LHC
- Sensitivity to top-Yukawa coupling
- Moderate K-factors (NLO corrections)

NLO QCD corrections available,
scale uncertainty drastically reduced

scale: $\mu_0 = m_t + m_H/2$

LHC: $K \sim 1.2$

Tevatron: $K \sim 0.8$



Beenakker, Dittmaier, Krämer, Plümper, Spira, Zerwas (2001)
Dawson, Reina, Wackerroth, Orr, Jackson (2001, 2003)