# Problem set for the lecture <br> Particle Detectors, WS 2015/16 

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Problem set 6
Deadline: Thursday December 3, 10am
(Please drop into mailbox number 1 on the ground floor of the Gustav-Mie building.)

1. Momentum measurement in a cylindrical detector

A cylindrical detector is built around the point of interaction of a storage ring experiment, with a solenoid field along the z-axis of the cylinder. All particles originate in the point of interaction.
(a) What is, using classical mechanics, the transverse momentum of a charged particle, in $\mathrm{GeV} / \mathrm{c}$, depending on a measured radius of curvature $\rho$ in meters and the field strength $B$ in Tesla?
(b) The detector has a radius $R=3.5 \mathrm{~m}$ and the strength of the magnetic field is 1 T . What is the minimum transverse momentum for a particle to be able to leave the detector?
(c) Consider a muon being emitted perpendicular to the z-axis, with a momentum of either $5 \mathrm{GeV} / \mathrm{c}$ or $100 \mathrm{GeV} / \mathrm{c}$. What are the differences $\alpha$ in azimuth of the coordinate where the muon leaves the detector and the projection of its original direction? What are the respective lengths $L$ of the path through the detector? Hint: What is the central angle of the circular segment defined by $\rho$ and the chord $R$ ?
(d) The position of a particle within the detector can be measured at three thin layers of multiwire proportional chambers, at $R_{1}=1 \mathrm{~m}, R_{2}=2 \mathrm{~m}$ and $R_{3}=3 \mathrm{~m}$. The spatial resolution of the chambers along the azimuth direction, here defined as the maximum possible error in position measurement, is 1 mm . Again consider the two muon momenta from part c). What are the maximum relative errors on the momentum measurement of the muon? Please note that the initial direction of the muon is not known a priori but can be reconstructed from the position measurements together with the momentum.
Hint: The small angle approximation can be used for trigonometric functions.

## 2. Efficiency and fake rate

The efficiency of a tracking detector is measured to be $95 \%$, while its fake rate (the probability to have a signal without any incoming particles) is found to be $1 \%$. To further improve the efficiency, it is decided to stack three detector layers on top of each other.
(a) Calculate the efficiency for the cases of using a logical OR of all three layers to detect a particle, and for the logical AND of at least two layers.
(b) What happens to the fake rate in the two cases?
(2 Points)

## 3. NUMERICAL SIMULATION - Particle propagation in a magnetic field

Simulate the propagation of a charged particle perpendicular to a uniform magnetic field, at first ignoring all interactions with matter. Use the classical equations of motion, and solve them using a stepping algorithm, splitting the trajectory into small steps. Along each step assume the force to be constant in magnitude and direction and propagate the particle using a uniformly accelerated motion with parameters defined by the initial conditions.
(a) Propagate a pion with a momentum of $1 \mathrm{GeV} / \mathrm{c}$ until it has completed a full circle, with $B=1 \mathrm{~T}$. Due to the linear approximation of each step, it will typically not pass through its point of origin. Plot the radial difference as a function of number of cycles. Is there a trend? How does the error scale with respect to the length of the steps?
(b) Now assume that a pion has been emitted in the centre of a cylindrical detector with a radius of 3 m , perpendicular to a uniform magnetic field of $B=1 \mathrm{~T}$ which points along the $z$-axis of the cylinder. Choosing a reasonable step size, simulate the path of the pion for a wide range of momenta. What is its angular deflection, i.e. the difference in azimuth coordinate of the points of exit with and without magnetic field, depending on its momentum?
(c) Assume the detector to be filled with argon gas, and the energy loss of the pion being $d E / d x=3.6 \mathrm{keV} / \mathrm{cm}$. What is the change in angular deflection and radius of curvature at the points of exit due to the energy losses in the gas, depending on particle momentum?

