3. Towards Physics: Reconstruction and Kinematics

3.1 Event selection, Trigger

3.2 First results on the performance of the LHC detectors

3.3 Relativistic Kinematics (repetition from Particle Physics II)

3.4 Important variables for pp collisions
Erwartete Produktionsraten am LHC

- Inelastische Proton-Proton Reaktionen: 1 Milliarde / sec
- Quark - Quark/Gluon Streuungen mit ~100 Millionen / sec großen transversalen Impulsen
- b-Quark Paare: 5 Millionen / sec
- Top-Quark Paare: 8 / sec
- $W \rightarrow e \nu$: 150 / sec
- $Z \rightarrow e e$: 15 / sec
- Higgs (150 GeV): 0.2 / sec
- Gluino, Squarks (1 TeV): 0.03 / sec

Dominante harte Streuprozesse: Quark - Quark
Quark - Gluon
Gluon - Gluon
How to Select Interesting Events?

Bunch crossing rate: 40 MHz, ~20 interactions per BX ($10^9$ events/s)
- can only record ~300 event/s (1.5 MB each), still 450 MB/s data rate

Need highly efficient and highly selective TRIGGER
- raw event data (70 TB/s) are stored in pipeline until trigger decision

ATLAS trigger has 3 levels (CMS similar with 2 levels)
- Level-1: hardware, ~3 µs decision time, 40 MHz $\rightarrow$ 75 kHz
- Level-2: software, ~40 ms decision time, 75 kHz $\rightarrow$ 2 kHz
- Level-3: software, ~4 s decision time, 2 kHz $\rightarrow$ 300 Hz
ATLAS Trigger System

Main trigger objects: at Level 1:

- $e/\gamma$ clusters (calo)
- Muons (muon)
- Jets (high $p_T$, calo)
- Missing transverse energy (calo)
Trigger system selects
~200 “collisions” per sec.

LHC data volume per year:
10-15 Petabytes
= 10-15 \cdot 10^{15} \text{ Byte}
Actually recorded are raw data with ~400 MB/s for ATLAS and CMS

- mainly electronics numbers
  - e.g. number of a detector element where the ADC (Analog-to-Digital converter) measured a signal with x counts...
From Raw Data To Physics

We need to go from raw data back to physics

- reconstruction + analysis of the event(s)
Towards Physics:
some aspects of reconstruction of physics objects

• As discussed before, key signatures at Hadron Colliders are

Leptons:  
- $\mu$ (dedicated muon systems, combination of inner tracking and muon spectrometers)
- $\tau$ hadronic decays: $\tau \rightarrow \pi^+ + n \pi^0 + \nu$ (1 prong)  
  \hspace{1cm} \rightarrow \pi^+\pi^-\pi^+ + n \pi^0 + \nu$ (3 prong)

Photons:  
- $\gamma$ (tracking + very good electromagnetic calorimetry)

Jets:  
- electromagnetic and hadronic calorimeters

b-jets  
- identification of b-jets (b-tagging) important for many physics studies

Missing transverse energy: inferred from the measurement of the total energy in the calorimeters; needs understanding of all components… response of the calorimeter to low energy particles
Requirements on e/γ Identification in ATLAS/CMS

Electron identification

** Isolated electrons: e/jet separation
- \( R_{\text{jet}} \sim 10^5 \) needed in the range \( p_T > 20 \) GeV
- \( R_{\text{jet}} \sim 10^6 \) for a pure electron inclusive sample (\( \varepsilon_e \sim 60-70\% \))

** Soft electron identification – e/π separation
- B physics studies (J/ψ)
- Soft electron b-tagging (WH, ttH with \( H \rightarrow bb \))

Photon identification

** γ/jet and γ/π⁰ separation
- Main reducible background to \( H \rightarrow γγ \) comes from jet-jet and is \( \sim 2 \cdot 10^6 \) larger than signal
- \( R_{\text{jet}} \sim 5000 \) in the range \( E_T > 25 \) GeV
- \( R \) (isolated high-\( p_T \) π⁰) \( \sim 3 \)

** Identification of conversions
Jet reconstruction and energy measurement

- A jet is NOT a well defined object (fragmentation, gluon radiation, detector response)

- The detector response is different for particles interacting electromagnetically (e, γ) and for hadrons
  → for comparisons with theory, one needs to correct back the calorimeter energies to the „particle level“ (particle jet)

*Common ground between theory and experiment*

- One needs an algorithm to define a jet and to measure its energy
  conflicting requirements between experiment and theory (exp. simple, e.g. cone algorithm, vs. theoretically sound (no infrared divergencies))

- Energy corrections for losses of fragmentation products outside jet definition and underlying event or pileup energy inside
**Main corrections:**

- In general, calorimeters show different response to electrons/photons and hadrons

- Subtraction of offset energy not originating from the hard scattering (inside the same collision or pile-up contributions, use minimum bias data to extract this)

- Correction for jet energy out of cone (corrected with jet data + Monte Carlo simulations)
3.2 First results on the performance of the LHC Detectors
3.2 Detector Performance
## Detector Hardware Status in 2010

<table>
<thead>
<tr>
<th>Subdetector</th>
<th>Number of Channels</th>
<th>Operational Fraction</th>
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</thead>
<tbody>
<tr>
<td>Pixels</td>
<td>80 M</td>
<td>97.9%</td>
</tr>
<tr>
<td>SCT Silicon Strips</td>
<td>6.3 M</td>
<td>99.3%</td>
</tr>
<tr>
<td>TRT Transition Radiation Tracker</td>
<td>350 k</td>
<td>98.2%</td>
</tr>
<tr>
<td>LAr EM Calorimeter</td>
<td>170 k</td>
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<tr>
<td>Tile calorimeter</td>
<td>9800</td>
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<tr>
<td>Hadronic endcap LAr calorimeter</td>
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<tr>
<td>Forward LAr calorimeter</td>
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<td>100%</td>
</tr>
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<td>MDT Muon Drift Tubes</td>
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<tr>
<td>CSC Cathode Strip Chambers</td>
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<td>RPC Barrel Muon Trigger</td>
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<tr>
<td>TGC Endcap Muon Trigger</td>
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</tr>
<tr>
<td>LVL1 Calo trigger</td>
<td>7160</td>
<td>99.8%</td>
</tr>
</tbody>
</table>

Very small number of non-working detector channels (out of several millions) in both experiments
Tracking
(i) Inner Detector performance: hits, tracks, resonances,...

- Very good agreement for the average number of hits on tracks in the silicon pixel and strip detectors

- Material distribution in the inner detector is well described in Monte Carlo (nice cross-check with $K^0$-mass dependence on radius in the Monte Carlo)
(ii) How well can b-quarks be tagged?

- b quarks fragment into B hadrons (mesons and baryons)
- B mesons have a lifetime of ~1.5 ps
  They fly in the detector about 2-3 mm before they decay

→ reconstruction of a secondary vertex possible
  (requires high granularity silicon pixel and strip detectors close to the interaction point)
→ tracks from B meson decays have a large impact parameter w.r.t. the primary vertex
…. towards b-tagging

An example of a jet tagged with the secondary vertex tagger (SV0) (Light jet probability: $10^{-4}$)

- Decay length = 3.7 mm
- Decay length significance = 22
- Lifetime = 3.1 ps
- Vertex mass = 2.5 GeV
- Number of tracks = 5
.... CMS b-tagged candidate event

Primary Vertex

Secondary Vertex (2σ ellipse) with 4 attached tracks

All other tracks Pt > 500 MeV

CMS experiment at LHC, CERN
Run 124022 / Event 13598392
2009-12-12 00:26:16 CEST
Four Tracks Secondary Vertex
ATLAS results on b-tagging performance:

Distribution of the signed transverse impact parameter with respect to primary vertex for tracks of b-tagging quality associated to jets, for experimental data (solid black points) and for simulated data (filled histograms for the various flavors). The ratio data/simulation is shown at the bottom of the plot.

Light-jet rejection as a function of the b-jet tagging efficiency for the early tagging algorithms (JetProb and SV0) and for the high performance algorithms, based on simulated top-antitop events.
(iii) Some performance figures on electrons from 2011 data:

Electron ID efficiency in ATLAS
An example of a two-jet event reconstructed in ATLAS.
(iv) Some performance figures on jet-energy scale from 2011 data:

Jet energy scale, E-flow in CMS
Distribution of $E_T^{\text{miss}}$ as measured in a data sample of $Z \rightarrow \text{ee}$ events. The expectation from Monte Carlo simulation is superimposed (histogram) and normalized to data, after each Monte Carlo sample is weighted with its corresponding cross-section. The ratio of the data distribution and the Monte Carlo distribution is shown below the plot.

Resolution of $E_x^{\text{miss}}$ and $E_y^{\text{miss}}$ as a function of the total transverse energy in the event calculated by summing the $p_T$ of muons and the total calorimeter energy. The resolution in $Z \rightarrow \text{ee}$ and $Z \rightarrow \mu\mu$ events is compared with the resolution in minimum bias for data taken at $\sqrt{s} = 7$ TeV. The fit to the resolution in Monte Carlo minimum bias and $Z \rightarrow \text{ee}$ events are superposed.

$$\sigma(E_{x,y}^{\text{miss}}) = a \oplus b \sqrt{\sum E_T}$$
(vi) Muons

\[ p_T(\mu^-) = 27 \text{ GeV} \quad \eta(\mu^-) = 0.7 \]
\[ p_T(\mu^+) = 45 \text{ GeV} \quad \eta(\mu^+) = 2.2 \]
\[ M_{\mu\mu} = 87 \text{ GeV} \]

\textit{Z} \rightarrow \mu\mu \text{ candidate in 7 TeV collisions}
Run: 152845, Event: 3338173
Date: 2010-04-12 16:56:44 CEST

\[ p_T(\mu^-) = 40 \text{ GeV} \]
\[ \eta(\mu^-) = 2.0 \]
\[ E_T^{\text{miss}} = 41 \text{ GeV} \]
\[ M_T = 83 \text{ GeV} \]

**W\rightarrow\mu\nu** candidate in 7 TeV collisions
$\mu^+\mu^-$ mass spectrum

Well known resonances. Observed widths depend on $p_T$ resolution.
Again, check for biases in mass value as a function of $\eta$, $\phi$, $p_T$ ...
3.3 Relativistic Kinematics

Throughout this section, natural units are used, i.e. $\hbar = c = 1$.

The following conversions are useful: $\hbar c = 197.3$ MeV fm
$(\hbar c)^2 = 0.3894$ (GeV)$^2$
Lorentz Transformations

4-vector $p = (E, \vec{p})$, $p^2 = E^2 - |\vec{p}|^2 = m^2$

velocity of the particle $\beta = |\vec{p}| / E$

$(E^*, \vec{p}^*)$ viewed from a frame moving with velocity $\beta$

\[
\begin{pmatrix}
E^* \\
\rho_{\perp}^ *
\end{pmatrix} =
\begin{pmatrix}
\gamma & -\gamma \beta \\
-\gamma \beta & \gamma
\end{pmatrix}
\begin{pmatrix}
E \\
\rho_{\parallel}
\end{pmatrix},
\rho_T^* = \rho_T,
\gamma = \frac{1}{\sqrt{1 - \beta^2}}
\]

where $\rho_T(\rho_{\parallel})$ are the components of $\vec{p}$ perpendicular (parallel) to $\beta$
Other 4-vectors transform in the same way:

e.g. space-time vectors \(\mathbf{x} = (t, \mathbf{x})\)

Scalar products of four-vectors are Lorentz invariant, independent of the reference frame:

\[
p_1 \cdot p_2 = E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2
\]

Therefore quantities like cross sections are expressed in terms of scalar products of four-vectors.
Centre-of-mass energy

• In the collision of two particles with masses \( m_1 \) and \( m_2 \) the total centre-of-mass energy can be expressed in the Lorentz-invariant form:

\[
E_{cm} = \left[ \left( E_1 + E_2 \right)^2 - \left( \mathbf{p}_1 + \mathbf{p}_2 \right)^2 \right]^{1/2},
\]

\[
= \left[ m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1 \beta_2 \cos \theta) \right]^{1/2}
\]

where \( \theta \) is the angle between the particles.
Laboratory Frame

In the laboratory frame, one of the particles, e.g. particle 2, is at rest. The centre-of-mass energy is then given by:

$$E_{cm} = \left( m_1^2 + m_2^2 + 2E_{1lab} m_2 \right)^{1/2}$$

The velocity of the centre-of-mass system in the lab frame is:

$$\beta_{cm} = p_{lab} / \left( E_{1lab} + m_2 \right),$$

where \( p_{lab} = p_{1lab} \) and \( \gamma_{cm} = \left( E_{1lab} + m_2 \right) / E_{cm} \).

The centre-of-mass momenta of particles 1 and 2 are of magnitude

$$p_{cm} = p_{lab} \frac{m_2}{E_{cm}}.$$
Examples

• A beam of $K^+$ mesons with a momentum of 800 MeV hits a proton target at rest.

$m_K = 493.7 \text{ MeV}, \ m_p = 938 \text{ MeV}, \ p_K = 0.80 \text{ GeV}$

Then the centre-of-mass energy is calculated to be: $E_{cm} = 1.699 \text{ GeV}$  
$p_{cm} = 0.442 \text{ GeV}$

• At the LHC protons collide in their centre-of-mass system with a centre-of-mass energy of 14 TeV.

This corresponds to an energy of an incoming proton in a fixed target experiment (protons on protons) of $\sim 10^{17} \text{ GeV}$

(such energies can only be reached in cosmic rays! but flux is not high enough to produce large numbers of interesting particles)
Comparison with cosmic rays

E spectrum falls as $E^{-2.7}$ to knee at $E \approx 5 \times 10^{15}$ eV
$= 5 \times 10^6$ GeV
$\sim 1$ particle/m² and year
origin galactic

above $\sim E^{-3}$

back to $E^{-2.7}$ at very
highest energies

conversion to $E_{cm}$

<table>
<thead>
<tr>
<th>$E_b$ [eV]</th>
<th>$E_{cm}$ [TeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{13}$</td>
<td>0.137</td>
</tr>
<tr>
<td>$10^{15}$</td>
<td>1.370</td>
</tr>
<tr>
<td>$10^{17}$</td>
<td>13.70</td>
</tr>
<tr>
<td>$10^{19}$</td>
<td>137.0</td>
</tr>
<tr>
<td>$10^{21}$</td>
<td>1370.</td>
</tr>
</tbody>
</table>

$\Rightarrow$ existence of **very powerful cosmic accelerators**. How do they work?
GZK (Greisen-Zatsepin-Kuzmin) Limit

The sharp drop in the cosmic ray spectrum at $10^{20}$ eV is explained by interactions of protons with photons from cosmic background radiation

\[ \gamma_{\text{CMB}} + p \rightarrow \Delta^+ \rightarrow p + \pi \]

\[ E_\gamma = kT = 2.6 \cdot 10^{-4} \text{eV} \ (T = 3K) \]

\[ E_p = 1 \cdot 10^{20} \text{eV} \]

\[ E_{\text{cms}} \approx 1 \text{GeV} \]

At CMS energies around 1 GeV the cross sections for $\pi$-production through the $\Delta$-resonance becomes large. Thus protons loose energy.

Cosmic protons at this energy have a mean free path of 160 MLY (GZK horizon). Thus extragalactic protons with energies larger than $10^{20}$ eV should not reach the earth. Recent measurements of the Auger experiment confirm this cut-off.

Auger Experiment

The combined energy spectrum is dotted with two functions and compared to data from the HiRes instrument. The systematic uncertainty of the flux scaled by $E^3$ due to the uncertainty of the energy scale of 22% is indicated by arrows.
The matrix elements for the scattering or decay process are written in terms of an invariant amplitude $-i\, M$. As an example, the S-matrix for $2\rightarrow 2$ scattering is related to $M$ by

$$
\langle p_1^\prime p_2^\prime \mid S \mid p_1 p_2 \rangle = I - i(2\pi)^4 \delta^4 (p_1 + p_2 - p_1^\prime - p_2^\prime) \\
\times \frac{M(p_1, p_2; p_1^\prime, p_2^\prime)}{(2E_1)^{1/2} (2E_2)^{1/2} (2E_1^\prime)^{1/2} (2E_2^\prime)^{1/2}}
$$

The normalization is such that

$$
\langle p^\prime \mid p \rangle = (2\pi)^3 \delta^3 (p - p^\prime)
$$

The task is to calculate the invariant amplitude $M$ for a given physics process. In particle physics this is achieved using the Feynman calculus (see lecture on Particle Physics II).
The partial decay rate of a particle of mass $m$ into $n$ bodies in its rest frame is given in terms of the Lorentz-invariant matrix element $M$ by

$$d\Gamma = \frac{(2\pi)^4}{2m} |M|^2 d\Phi_n (P; p_1, \ldots, p_n)$$

where $d\Phi_n$ is an element of $n$-body phase space given by:

$$d\Phi_n (P; p_1, \ldots, p_n) = \delta^4(P - \sum_{i=1}^n p_i) \prod_{i=1}^n \frac{d^3p_i}{(2\pi)^3 2E_i}$$
Survival probability of Decay

If a particle of mass $m$ has a mean proper lifetime of $\tau (=1/\Gamma)$ and an energy-momentum 4-vector of $(E,p)$, then the probability that it lives for a time $t$ or greater before decaying is given by

$$P(t) = e^{-t \frac{\Gamma}{\gamma}} = e^{-mt \frac{\Gamma}{E}}$$

and the probability that it travels a distance $x$ or greater is

$$P(x) = e^{-mx \frac{\Gamma}{|p|}}$$
Example (i): Two-Body Decay

In the rest frame of a particle of mass $m$, decaying into two particles labelled 1 and 2

$$E_1 = \frac{m^2 - m_2^2 + m_1^2}{2m},$$

$$d\Gamma = \frac{1}{32\pi^2} |M|^2 \frac{|p_1|}{m^2} d\Omega,$$

where $d\Omega = d\phi_1 d(cos \theta_1)$ is the solid angle of particle 1.
The invariant mass $m$ of the mother particle in a two-body decay is given by $m = E_{cm}$ using the previous formula:

$$E_{cm} = \left[ (E_1 + E_2)^2 - (p_1 + p_2)^2 \right]^{1/2}$$

$$= \left[ m_1^2 + m_2^2 + 2E_1E_2(1 - \beta_1\beta_2 \cos \theta) \right]^{1/2}$$

Generalisation: the invariant mass of $n$ particles is given by:

$$m = (p_1 + p_2 + p_3 + \ldots + p_n)^2$$
Example (ii): Three-Body Decay

Defining $p_{ij} = p_i + p_j$ and $m^2_{ij} = p^2_{ij}$ then

$$m^2_{12} + m^2_{23} + m^2_{13} = m^2 + m^2_1 + m^2_2 + m^2_3$$

and $m^2_{12} = (P - p_3)^2 = m^2 + m^2_3 - 2 m E_3$

$E_3$ is the energy of particle 3 in the rest frame of $m$.

In that frame, the momenta of the three decay particles lie in a plane.
The relative orientation of these three momenta is fixed if their energies are known. The momenta can therefore be specified in space by giving three Euler angles $(\alpha, \beta, \gamma)$ that specify the orientation of the final system relative to the initial particle.

\[
d\Gamma = \frac{1}{(2\pi)^5} \frac{1}{16M} |\mathcal{M}|^2 dE_1 dE_2 d\alpha d(\cos\beta) d\gamma
\]

Alternatively

\[
d\Gamma = \frac{1}{(2\pi)^5} \frac{1}{16M^2} |\mathcal{M}|^2 \frac{1}{|p_1^*||p_3|} dm_{12} d\Omega_1^* d\Omega_3
\]

where $(|p_1^*|, \Omega_1^*)$ is the momentum of particle 1 in the rest frame of 1 and 2, and $\Omega_3$ is the angle of particle 3 in the rest frame of the decaying particle.
Three-Body Decay (cont.)

\[ |\mathbf{p}_1^*| \text{ and } |\mathbf{p}_3| \text{ are given by} \]

\[ |\mathbf{p}_1^*| = \frac{\left[ (m_{12}^2 - (m_1 + m_2)^2)(m_{12}^2 - (m_1 - m_2)^2) \right]^{1/2}}{2m_{12}} \]

\[ |\mathbf{p}_3| = \frac{\left[ (m^2 - (m_{12} + m_3)^2)(m^2 - (m_{12} - m_3)^2) \right]^{1/2}}{2m} \]
Particles participating in sequential two-body decay chain. Particles labeled 1 and 2 are visible while the particle terminating the chain (a) is invisible.

\[
(m_{12}^{\text{max}})^2 = \frac{(m_c^2 - m_b^2)(m_b^2 - m_a^2)}{m_b^2}, \quad \text{provided particles 1 and 2 are massless.}
\]

\[
(m_{12}^{\text{max}})^2 = m_1^2 + \frac{(m_c^2 - m_b^2)}{2m_b^2} \times (m_1^2 + m_b^2 - m_a^2 + \sqrt{(-m_1^2 + m_b^2 - m_a^2)^2 - 4m_1^2m_a^2}).
\]

If visible particle 1 has non-zero mass \( m_1 \).
Differential Cross Section

In the rest frame of $m_2$ (lab)

\[
\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} = m_2 p_{1\text{lab}}
\]

In the centre-of-mass frame

\[
\sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2} = p_{1\text{cm}} \sqrt{s}
\]

\[
d\sigma = \frac{(2\pi)^4 |\mathcal{M}|^2}{4 \sqrt{(p_1 \cdot p_2)^2 - m_1^2 m_2^2}} \times d\Phi_n(p_1 + p_2; p_3, \ldots, p_{n+2})
\]
Mandelstam Variables (two-to-two process)

\[ s = (p_1 + p_2)^2 = (p_3 + p_4)^2 \]
\[ = m_1^2 + 2E_1E_2 - 2p_1 \cdot p_2 + m_2^2, \]
\[ t = (p_1 - p_3)^2 = (p_2 - p_4)^2 \]
\[ = m_1^2 - 2E_1E_3 + 2p_1 \cdot p_3 + m_3^2, \]
\[ u = (p_1 - p_4)^2 = (p_2 - p_3)^2 \]
\[ = m_1^2 - 2E_1E_4 + 2p_1 \cdot p_4 + m_4^2, \]
\[ s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2. \]
Using the relations given above, the two-body cross section can be written as:

\[
\frac{d\sigma}{dt} = \frac{1}{64\pi s} \frac{1}{|p_{1\text{cm}}|^2} |M|^2
\]

Advantage to use Lorentz invariant quantities, like \( t \).
The variable $t$ is given by:

$$t = (E_{1\text{cm}} - E_{3\text{cm}})^2 - (p_{1\text{cm}} - p_{3\text{cm}})^2 - 4p_{1\text{cm}}p_{3\text{cm}} \sin^2(\theta_{\text{cm}}/2)$$

$$= t_0 - 4p_{1\text{cm}}p_{3\text{cm}} \sin^2(\theta_{\text{cm}}/2)$$

where $\theta_{\text{cm}}$ is the angle between particle 1 and 3.

The limiting values $t_0$ ($\theta_{\text{cm}} = 0$) and $t_1$ ($\theta_{\text{cm}} = \pi$) for $2 \rightarrow 2$ scattering are

$$t_0(t_1) = \left[\frac{m_1^2 - m_3^2 - m_2^2 + m_4^2}{2\sqrt{s}}\right]^2 - (p_{1\text{cm}} \mp p_{3\text{cm}})^2$$
The centre-of-mass energies and momenta of the incoming particles are

\[ E_{1\text{cm}} = \frac{s + m_1^2 - m_2^2}{2\sqrt{s}} , \quad E_{2\text{cm}} = \frac{s + m_2^2 - m_1^2}{2\sqrt{s}} \]

For \( E_{3\text{cm}} \) and \( E_{4\text{cm}} \), change \( m_1 \) to \( m_3 \) and \( m_2 \) to \( m_4 \) (same particles).

\[ p_{i\text{cm}} = \sqrt{E_{i\text{cm}}^2 - m_i^2} \text{ and } p_{1\text{cm}} = \frac{p_{1\text{lab}} m_2}{\sqrt{s}} \]

Here the subscript lab refers to the frame where particle 2 is at rest.
3.4 Important kinematic Variables in pp collisions
(i) Rapidity $y$

Usually the beam direction is defined as the $z$ axis (Transverse plane: $x$-$y$ plane).

The rapidity $y$ is defined as:

$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) = \tanh^{-1} \left( \frac{p_z}{E} \right)$$

Under a Lorentz boost in the $z$-direction to a frame with velocity $\beta$

the rapidity $y$ transforms as:

$$y \rightarrow y - \tanh^{-1} \beta$$

Hence the shape of the rapidity distribution $dN/dy$ is invariant, as are differences in rapidity.
(ii) Pseudorapidity $\eta$

Rapidity:

$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) = \tanh^{-1} \left( \frac{p_z}{E} \right)$$

For $p \gg m$, the rapidity may be expanded to obtain

$$y = \frac{1}{2} \ln \frac{\cos^2(\theta/2) + m^2/4p^2 + \ldots}{\sin^2(\theta/2) + m^2/4p^2 + \ldots}$$

$$\approx -\ln \tan(\theta/2) \equiv \eta$$

where $\cos \theta = p_z/p$.

Identities:

$$\sinh \eta = \cot \theta, \quad \cosh \eta = 1/\sin \theta, \quad \tanh \eta = \cos \theta$$
Relation between pseudorapidity $\eta$ and polar angle $\theta$

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<th>$\eta$</th>
<th>$\theta$</th>
</tr>
</thead>
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<tr>
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</tr>
<tr>
<td>2</td>
<td>15.41</td>
</tr>
<tr>
<td>3</td>
<td>5.70</td>
</tr>
<tr>
<td>4</td>
<td>2.10</td>
</tr>
</tbody>
</table>

$\theta = \frac{360}{\pi} \arctan e^{-\eta}$
(iii) Distance in $\eta - \phi$ space:

Rapidity $y$: 
$$y = \frac{1}{2} \ln \left[ \frac{(E + p_z)}{(E - p_z)} \right]$$

Pseudorapidity $\eta$: 
$$\eta = -\ln \tan(\theta/2)$$

Distance in $\eta$-$\phi$: 
$$\Delta R = \sqrt{\Delta \eta^2 + \Delta \phi^2}$$
(iv) Invariant cross section

The invariant cross section may also be rewritten

\[
E \frac{d^3 \sigma}{d^3 p} = \frac{d^3 \sigma}{d \phi \, dy \, p_T \, dp_T} \implies \frac{d^2 \sigma}{\pi \, dy \, d(p_T^2)}
\]

The second form is obtained using the identity \(dy/dp_z = 1/E\).

The third form represents the average over \(\phi\).
(v) Transverse Energy

At hadron colliders, a significant and unknown proportion of the energy of the incoming hadrons in each event escapes down the beam-pipe. Consequently if invisible particles are created in the final state, their net momentum can only be constrained in the plane transverse to the beam direction. Defining the z-axis as the beam direction, this net momentum is equal to the missing transverse energy vector

$$E_T^{\text{miss}} = - \sum_i p_T(i)$$

where the sum runs over the transverse momenta of all visible final state particles.
(vi) Momenta of invisible particles

Consider a single heavy particle of mass $M$ produced in association with visible particles which decays to two particles, of which one (labelled particle 1) is invisible. The mass of the parent particle can be constrained with the quantity $M_T$ defined by

\[ M_T^2 \equiv [E_T(1) + E_T(2)]^2 - [\mathbf{p}_T(1) + \mathbf{p}_T(2)]^2 \]
\[ = m_1^2 + m_2^2 + 2[E_T(1)E_T(2) - \mathbf{p}_T(1) \cdot \mathbf{p}_T(2)] \]

where

\[ \mathbf{p}_T(1) = E_T^{\text{miss}} \]

This quantity is called the transverse mass.
Transverse mass

\[ M_T^2 \equiv [E_T(1) + E_T(2)]^2 - [p_T(1) + p_T(2)]^2 = m_1^2 + m_2^2 + 2[E_T(1)E_T(2) - p_T(1) \cdot p_T(2)] \]

where \( p_T(1) = E_T^{\text{miss}} \)

The distribution of event \( M_T \) values possesses an end-point at

\[ M_T^{\text{max}} = \bar{M}. \]

If \( m_1 = m_2 = 0 \)

\[ M_T^2 = 2|p_T(1)||p_T(2)|(1 - \cos \phi_{12}) \]

where \( \phi_{ij} \) is defined as the angle between particles \( i \) and \( j \) in the transverse plane.
Example: Transverse mass of the W boson

\[ m_T = \sqrt{2P_T(e)E_T^{\text{miss}}(1 - \cos \Delta \phi)} \]

(see previous slide)