9. Supersymmetry

9.1 Introduction, concept

9.2 Motivation for SUSY

9.3 Breaking of Supersymmetry

9.4 Supersymmetry pre-LHC

9.5 Search for supersymmetry at the LHC

9.6 How can the parameters of a SUSY model be constrained
Why Physics Beyond the Standard Model?

1. Gravity is not yet incorporated in the Standard Model

2. Dark Matter not accommodated

3. Many open questions in the Standard Model
   - Hierarchy problem: $m_W (100 \text{ GeV}) \rightarrow m_{\text{Planck}} (10^{19} \text{ GeV})$
   - Unification of couplings
   - Flavour / family problem
   - ...

All this calls for a more fundamental theory of which the Standard Model is a low energy approximation $\rightarrow$ New Physics

Candidate theories: Supersymmetry, Extra Dimensions, Technicolor

Many extensions predict new physics at the TeV scale !!

Strong motivation for LHC, mass reach $\sim 3 \text{ TeV}$
9.1 Introduction: what is Supersymmetry (SUSY)?

SUSY is an extension of the Standard Model (since ~ 1970) that introduces a new symmetry between fermions and bosons:

\[ \text{Spin-} \frac{1}{2} \text{ matter particles (fermions)} \iff \text{Spin-} 1 \text{ force particles (bosons)} \]

SUSY transformation (operator Q):
\[
\begin{align*}
Q \left| \text{Fermion} \right\rangle & \sim \left| \text{Boson} \right\rangle \\
Q \left| \text{Boson} \right\rangle & \sim \left| \text{Fermion} \right\rangle
\end{align*}
\]

\( \rightarrow \) SUSY doubles the number of particles

SUSY is a symmetry, not a theory. A model (a Lagrangian) can possess supersymmetry as a property.
Supersymmetry

Extends the Standard Model by predicting a new symmetry
Spin $\frac{1}{2}$ matter particles (fermions) $\iff$ Spin 1 force carriers (bosons)

Standard Model particles

SUSY particles

Standard Model and supersymmetry partners can be distinguished by a discrete Quantum number: R-parity:

$$R = (-1)^{3B+L+2S} = +1 \text{ SM particles}$$

$$-1 \text{ SUSY particles}$$
<table>
<thead>
<tr>
<th>Particle</th>
<th>Sparticle (corresp. SUSY particle)</th>
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</thead>
<tbody>
<tr>
<td>Spin-1/2</td>
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<td>quarks (L&amp;R)</td>
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<td>leptons (L&amp;R)</td>
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<tr>
<td>Spin-1</td>
<td>$B$, $W^0$</td>
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</tbody>
</table>

- **Particles**: quarks, leptons, neutrinos
- **Sparticles (corresponding SUSY particles)**: squarks, sleptons, sneutrinos
- **Spin-1**: $B$, $W^0$, $\gamma$, $Z^0$, $W^\pm$, gluon
- **Spin-0**:
<table>
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<td></td>
<td><strong>W^0</strong></td>
<td><strong>Wino^0</strong></td>
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<td><strong>Z^0</strong></td>
<td><strong>Wino^-</strong></td>
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<td><strong>W^±</strong></td>
<td><strong>gluino</strong></td>
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<td><strong>gluon</strong></td>
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</table>
Extended Higgs sector: 2 complex Higgs doublets

→ Degrees of freedom: 8 - 3 (Goldstone bosons) = 5 Higgs bosons: $h^0, H^0, A^0, H^±$
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<td>Spin-1</td>
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<tr>
<td>Higgs</td>
<td>Higgsinos</td>
</tr>
<tr>
<td>(H_1^1) ((H_1^2))</td>
<td>(\tilde{H}_1^1) ((\tilde{H}_1^2))</td>
</tr>
<tr>
<td>(H_2^1) ((H_2^2))</td>
<td>(\tilde{H}_2^1) ((\tilde{H}_2^2))</td>
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<tr>
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**Extended Higgs sector:** 2 complex Higgs doublets

\[\text{Degrees of freedom: } 8 - 3 \text{ (Goldstone bosons)} = 5 \text{ Higgs bosons:}\]

\[h^0, H^0, A^0, H^±\]
### Extended Higgs sector:

- 2 complex Higgs doublets
- Degrees of freedom: 8 - 3 (Goldstone bosons) = 5 Higgs bosons: \( h^0, H^0, A^0, H^\pm \)

### Table: Particles and Their Corresponding Sparticles

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<tr>
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<td>Bino</td>
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<td></td>
<td>Wino(^0)</td>
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<td></td>
<td>Wino(^\pm)</td>
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<td>gluino</td>
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<td>Spin-1</td>
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<td></td>
<td>( Z^0 )</td>
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<td></td>
<td>( W^\pm )</td>
</tr>
<tr>
<td></td>
<td>gluon</td>
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### Diagram:

- After Mixing
- 4 neutralinos
- Spin-1/2
- Spin-0
Extended Higgs sector: 2 complex Higgs doublets

→ Degrees of freedom: $8 - 3$ (Goldstone bosons) = 5 Higgs bosons: $h^0, H^0, A^0, H^\pm$

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<td>neutrinos (L)</td>
<td>sneutrinos (L)</td>
</tr>
<tr>
<td>B, $W^0$</td>
<td>Bino, $Wino^0$</td>
</tr>
<tr>
<td>$Z^0, W^\pm$</td>
<td>$Wino^\pm, gluino$</td>
</tr>
<tr>
<td>gluon</td>
<td></td>
</tr>
</tbody>
</table>

| Spin-0            |                                   |
| Higgs             | Higgsinos                         |
| $(H_1^1, H_1^2)$  | $(\tilde{H}_1^1, \tilde{H}_1^2)$ |
| $(H_2^1, H_2^2)$  | $(\tilde{H}_2^1, \tilde{H}_2^2)$ |

After Mixing

- 4 neutralinos
- 2 charginos

- 2 charginos
• Physical neutralinos and charginos are mixtures of Wino, Bino, Higgsinos

• Charginos:

\[
\begin{pmatrix}
\chi_1^+ \\
\chi_2^+
\end{pmatrix} =
\begin{pmatrix}
M_2 & \sqrt{2}m_W \sin \beta \\
\sqrt{2}m_W \cos \beta & \mu
\end{pmatrix}
\begin{pmatrix}
\tilde{W}^+ \\
\tilde{H}^+
\end{pmatrix}
\]

• Neutralinos:

\[
\begin{pmatrix}
\chi_1^0 \\
\chi_2^0 \\
\chi_3^0 \\
\chi_4^0
\end{pmatrix} =
\begin{pmatrix}
M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\
0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\
-m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\
m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0
\end{pmatrix}
\begin{pmatrix}
\tilde{B} \\
\tilde{W}^3 \\
\tilde{H}_1^0 \\
\tilde{H}_2^0
\end{pmatrix}
\]

Mass eigenstates depend on:

- \(M_1, M_2, \tan \beta, \mu\) SUSY masses and breaking parameters
- \(m_Z, \sin^2 \theta_W\) EWSB (mixing: \(B^0, W^0 \rightarrow Z, \gamma\))
Supersymmetric particles carry the same quantum numbers (hypercharge, weak isospin, color) of the standard model partners

That means they have the same interaction vertices

<table>
<thead>
<tr>
<th>Names</th>
<th>spin 0</th>
<th>spin 1/2</th>
<th>SU(3)_C, SU(2)_L, U(1)_Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>squarks, quarks</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(×3 families)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>(\tilde{u}_L \tilde{d}_L)</td>
<td>(u_L d_L)</td>
<td>(3, 2, \frac{1}{6})</td>
</tr>
<tr>
<td>\bar{u}</td>
<td>\tilde{u}_R</td>
<td>u_R^\dagger</td>
<td>(\bar{3}, 1, -\frac{2}{3})</td>
</tr>
<tr>
<td>\bar{d}</td>
<td>\tilde{d}_R^*</td>
<td>d_R^\dagger</td>
<td>(\bar{3}, 1, \frac{1}{3})</td>
</tr>
<tr>
<td>sleptons, leptons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(×3 families)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>(\tilde{\nu} \tilde{e}_L)</td>
<td>(\nu e_L)</td>
<td>(1, 2, -\frac{1}{2})</td>
</tr>
<tr>
<td>\bar{e}</td>
<td>\tilde{e}_R^*</td>
<td>e_R^\dagger</td>
<td>(1, 1, 1)</td>
</tr>
<tr>
<td>Higgs, higgsinos</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H_u</td>
<td>(H_u^+ H_u^0)</td>
<td>(\tilde{H}_u^+ \tilde{H}_u^0)</td>
<td>(1, 2, +\frac{1}{2})</td>
</tr>
<tr>
<td>H_d</td>
<td>(H_d^0 H_d^-)</td>
<td>(\tilde{H}_d^0 \tilde{H}_d^-)</td>
<td>(1, 2, -\frac{1}{2})</td>
</tr>
</tbody>
</table>

Table 1.1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions.
<table>
<thead>
<tr>
<th>Names</th>
<th>spin 1/2</th>
<th>spin 1</th>
<th>$SU(3)_C$, $SU(2)_L$, $U(1)_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>gluino, gluon</td>
<td>$\tilde{g}$</td>
<td>$g$</td>
<td>$(8, 1, 0)$</td>
</tr>
<tr>
<td>winos, W bosons</td>
<td>$\tilde{W}^{\pm}$ $\tilde{W}^0$</td>
<td>$W^{\pm}$ $W^0$</td>
<td>$(1, 3, 0)$</td>
</tr>
<tr>
<td>bino, B boson</td>
<td>$\tilde{B}^0$</td>
<td>$B^0$</td>
<td>$(1, 1, 0)$</td>
</tr>
</tbody>
</table>

Gauge supermultiplets in the Minimal Supersymmetric Standard Model.
R parity

- New terms in Lagrangian:
  \[ W_{RPV} = \frac{1}{2} (\lambda_{111} LL + \lambda'_{1} LQD + \lambda''_{1} UDD) + \mu LH \]
  - L-violating
  - B-violating
  - L-violating

Problem: These couplings lead to proton decay

Unacceptably high rate compared to experimental limits
(proton lifetime > 10^{33} years)

→ Strong limits on product of couplings

- Introduce multiplicative quantum number:
  \[ R_p = (-1)^{3(B-L)+2S} \]
  - +1 for SM particles
  - -1 for SUSY particles

- Impose \( R_p \) conservation:
  - Sparticles produced in pairs
  - Lightest SUSY particle (LSP) stable
Experimental consequences of R-parity conservation:

• SUSY particles are produced in pairs

• Lightest Supersymmetric Particle (LSP) is stable.

  LSP is only weakly interacting:
  LSP = χ^0_1 (lightest neutralino, in many models)

  → LSP behaves like a ν → it escapes detection

  → E_T^{miss} (typical SUSY signature)
**SUSY interactions, some examples**

The coupling constants are the same as in SM (strong, electroweak)

“Recipe” : Obtain SUSY interactions by exchanging
at a vertex two SM legs by corresponding SUSY legs
9.2 Motivation for supersymmetry

or what problems of the Standard Model does it solve?
(i) The Hierarchy or naturalness problem

- In the Standard Model, quadratically divergent quantum corrections to the Higgs mass appear

\[ \Delta m_{H}^{2} = -\frac{|\lambda_{f}|^{2}}{8\pi^{2}} \Lambda_{UV}^{2} + \ldots. \]

- The symmetry between bosons and fermions, which contribute with different sign (statistics), can cure this problem:

\[ \Delta m_{H}^{2} = \frac{\lambda_{S}}{16\pi^{2}} \left[ \Lambda_{UV}^{2} - 2m_{S}^{2} \ln(\Lambda_{UV}/m_{S}) + \ldots \right] \]

\[ \Delta m_{H} = f(m_{B}^{2} - m_{f}^{2}) \]

→ terms cancel one-by-one if SUSY were a perfect symmetry (i.e. if \( m(\text{particle}) = m(\text{sparticle}) \)). Since this is not the case, sparticles should not be too heavy (\( m_{\text{SUSY}} <\sim 1 \text{ TeV} \)).
Due to quantum corrections, e.g.

slope is changed due to contributions from SUSY particles.
(ii) SUSY provides a candidate for Dark Matter in the universe

Evidence from:

- Rotational curves of galaxies
- Gravitational lensing
- Cosmic microwave background (CMB)
Dark-Matter properties:

- Gravitationally interacting

- Not short-lived

- Not hot

- Not baryonic, no el. magnetic interaction

→ Unambiguous evidence for new physics!
• SUSY has a weakly interacting massive particle (WIMP), if R-parity is conserved: the lightest supersymmetric particle
• LSP = lightest neutralino, gravitino (depending on SUSY model)
9.3 Breaking of Supersymmetry

- or the dark side of SUSY -
• **Supersymmetry cannot be an exact symmetry**, since we have not seen SUSY particles with masses equal to the Standard Model particles (e.g. no scalar electrons with masses of 511 keV)

• It is expected that supersymmetry is **broken spontaneously**, i.e. the underlying model should have a Lagrangian density that is invariant under supersymmetry, but a vacuum state that is not.

• **The mechanism of SUSY breaking is not known**

• Our ignorance is parametrized by introducing extra terms that break SUSY explicitly (so called soft SUSY breaking terms*) into the Lagrangian and consider it as an “effective” Lagrangian

• → about 105 parameters are introduced: masses, couplings, mixing angles.....

*) only such terms are allowed that guarantee that no new quadratically divergent radiative corrections appear
A general parametrization comprises:

- Scalar mass terms: $m_{0}^{2} \phi_{i}^{2}$
- Gaugino mass terms: $\frac{1}{2} M_{a} V_{a}^{T} C V_{a}$
- Coupling terms of scalar particles: $A_{\lambda} \lambda \phi^{3}$, $B_{\mu} \mu \phi^{2}$

It is often assumed that these supersymmetry breaking terms originate at some high scale (Grand Unification scale (GUT scale), or gravity scale) .... maybe linked to some supergravity or superstring mechanism.

It is furthermore often assumed that the soft supersymmetry-breaking terms are universal at the GUT or supergravity scale.
The Minimal Supersymmetric Extension to the SM (MSSM)

It includes 105 additional parameters on top of the SM ones:

- SUSY violating mass terms for each partner of the SM fermions
- SUSY violating mass terms for each partner of the SM bosons
- Additional parameters on the Higgs sector (5 Higgs bosons in total)
- Trilinear couplings of all scalar particles

These parameters cannot be all independent:

- Or we would have large FCNC, CP violation, etc.
- Consistency with existing measurements indicate some ordering principle is at play
Bottom up approach - The phenomenological MSSM (pMSSM)

- Reasonable assumptions (motivated by current experimental results):
  - The mass matrices are diagonal and proportional to the identity matrix (no FCNC)
  - The trilinear couplings are proportional to the Yukawa couplings
  - No additional complex phase in the matrices (no additional CP violation)
  - Unified masses of first and second sfermions generations (from $k^0$-anti-$k^0$ mixing)

Then one ends up with the so-called phenomenological MSSM, which has 19 parameters in addition to the SM parameters

\[
\tan \beta : \text{the ratio of the vev of the two-Higgs doublet fields.}
\]

\[
M_A : \text{the mass of the pseudoscalar Higgs boson}
\]

\[
\mu : \text{the Higgs-higgsino mass parameter}
\]

\[
M_1, M_2, M_3 : \text{the bino, wino and gluino mass parameters.}
\]

\[
m_g, m_{\tilde{e}_R}, m_{\tilde{d}_R}, m_{\tilde{u}_L}, m_{\tilde{d}_L} : \text{first/second generation sfermion masses}
\]

\[
m_Q, m_{\tilde{L}_R}, m_{\tilde{e}_R}, m_L, m_{\tilde{\tau}_R} : \text{third generation sfermion masses}
\]

\[
A_t, A_b, A_\tau : \text{third generation trilinear couplings.}
\]
Top-down approaches: Constrained MSSM models

It is often assumed that these supersymmetry breaking terms originate at some high scale (Grand Unification scale (GUT scale), or gravity scale) .... maybe linked to some supergravity or superstring mechanism.

It is furthermore often assumed that the soft supersymmetry-breaking terms are universal at the GUT or supergravity scale.

The constrained MSSM models make strong assumptions about unification of SUSY parameters. They have been very popular for many years.

But they really suffer from the LHC constraints..... With time, they are becoming less and less popular.
If one assumes universality for all masses and couplings at the GUT scale, the following five parameters suffice to characterize the MSSM (Minimal Supersymmetric Standard Model)

**Five parameters:**

- \( m_0 = \) common sfermion mass at the GUT scale
- \( m_{1/2} = M1 = M2 = \mu \) (common gaugino mass at the GUT scale)
- \( A_0 \) common coupling term
- \( \tan \beta \) ratio of vacuum expectation value of the two Higgs doublets
- \( \mu \) Higgs mass term

The parameters \( \mu \) and \( \tan \beta \) are related to the Higgs sector of the MSSM and determine this sector –together with one Higgs mass, e.g. \( m_A \) –at tree level completely.

This model is referred to as “constrained MSSM”
The energy dependence of the SUSY particle masses can be calculated (renormalization group equations).

In such models, squarks and/or gluinos are the heaviest sparticles.

- Stops might be the lightest squarks.
- Mixing effects (see later) can lead to mass splitting between $t_L$ and $t_R$.
- Sleptons / charginos / neutralinos are lightest sparticles.

In this example a separate Higgs mass term is used at the GUT scale.
Example for a calculated mass spectrum at the electroweak scale:

- In many models, the lightest neutralino is the lightest SUSY particle (LSP)
- Lightest Higgs boson ($h^0$) might also be light, split from heavier Higgs particles
- Production of SUSY particles at the LHC is dominated by coloured squarks and gluinos
SUSY mass spectra for different SUSY breaking models:

- **mSUGRA**: minimal SUperGRAvity model (gravity responsible for SUSY breaking)
- **GMSB**: Gauge mediated SUSY Breaking (breaking via gauge interactions)
- **AMSB**: Anomaly mediated SUSY Breaking
Sparticle Masses and Mixing

(i) Sfermions:

- Each flavour of charged lepton or quark has both left- and right handed components, $f_{L,R}$ and these have separate spin-0 boson superpartners $f'_{LR}$.
- The superpartners can mix, and the mass matrix in MSSM is given by:

$$M_f^2 = \begin{pmatrix} m_{f_{LL}}^2 & m_{f_{LR}}^2 \\ m_{f_{LR}}^2 & m_{f_{RR}}^2 \end{pmatrix}$$

where the off-diagonal mixing terms take the general form (with $m_f$ being the corresponding fermion mass).

$$m_{f_{L,R}}^2 = m_f \left( A_f + \mu \tan \beta \right) \quad \text{for} \quad f = e, \mu, \tau, d, s, b$$

- Since mixing effects are proportional to the SM fermion masses, they are in particular important for third generation sfermions. They are as well enhanced for large $\tan \beta$.
- Diagonalization of this mass matrix leads to the physical SUSY particle masses. Mixing might split the two states in mass, usually referred to as the lighter $f'_1$ and the heavier state $f'_2$. 
Sparticle Masses and Mixing

(ii) Charginos:

Charginos are the supersymmetric partners of the $W^\pm$ and charged Higgs bosons $H^\pm$. Their masses are determined via the parameters: $M_2$, $\mu$ and $\tan \beta$, via the mass mixing matrix $M_C$:

$$-\frac{1}{2} (\tilde{W}^-, \tilde{H}^-) M_C \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}^+ \end{pmatrix} + \text{herm.conj.}$$

$$M_C \equiv \begin{pmatrix} M_2 & \sqrt{2} m_W \sin \beta \\ \sqrt{2} m_W \cos \beta & \mu \end{pmatrix}$$

where $M_2$ is the unmixed SU(2) gaugino mass and $\mu$ is the Higgs mass parameter.
Sparticle Masses and Mixing

(ii) Neutralinos:

- Masses of neutralinos, the supersymmetric partners of the neutral fields \((\tilde{W}^3, \tilde{B}, \tilde{H}^0_2, \tilde{H}^0_1)\)

are given by the following mixing matrix:

\[
m_N = \begin{pmatrix}
M_2 & 0 & -\frac{g_2 v_2}{\sqrt{2}} & \frac{g_2 v_1}{\sqrt{2}} \\
0 & M_1 & \frac{g' v_2}{\sqrt{2}} & -\frac{g' v_1}{\sqrt{2}} \\
-\frac{g_2 v_2}{\sqrt{2}} & \frac{g' v_2}{\sqrt{2}} & 0 & \mu \\
\frac{g_2 v_1}{\sqrt{2}} & -\frac{g' v_1}{\sqrt{2}} & \mu & 0 \\
\end{pmatrix}
\]

where \(M_1, M_2\) is the unmixed U(1), SU(2) gaugino mass, \(\mu\) is the Higgs mass parameter, and \(g_2\) and \(g'\) are the gauge couplings of the SU(2) / U(1) group.
Higgs boson and stop quark

This is to say: a Higgs mass of 126 GeV requires large stop masses and/or maximal mixing

But this directly challenges naturalness

\[ m_Z^2 = -2\mu^2 + 2 \frac{m_{H_d}^2 - \tan^2 \beta \ m_{H_u}^2}{\tan^2 \beta - 1} \]

\[
\max_{a_i} \left( \left| \frac{a_i}{m_Z^2} \frac{\partial m_Z^2(a_i)}{\partial a_i} \right| \right) < \Delta
\]

\[
\sqrt{m_{t_1}^2 + m_{t_2}^2} \lesssim 600 \text{ GeV} \frac{\sin \beta}{(1 + x_t^2)^{1/2}} \left( \frac{\log (\Lambda/ \text{TeV})}{3} \right)^{-1/2} \left( \frac{m_h}{120 \text{ GeV}} \right) \left( \frac{\Delta^{-1}}{20\%} \right)^{-1/2}
\]
Higgs boson and stop quark

Because of the large top Yukawa coupling, the Higgs boson and the stop sector turn out to be heavily connected

\[ m_h^2 = m_Z^2 \cos^2 \beta + \frac{3y_t^2m_t^2}{(4\pi)^2} \left[ \log \left( \frac{m_S^2}{m_t^2} \right) + X_t^2 \left( 1 - \frac{X_t^2}{12} \right) \right] + \cdots \]

The natural Higgs mass is \( m_Z \). However loop corrections are large

\[ X_t = (A_t - \frac{\mu}{\tan \beta}) \]

\[ M_S = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \]

\[ M_t^2 = \begin{bmatrix} m_Q^2 + m_t^2 & m_t(A_t - \mu^*/\tan \beta) \\ m_t(A_t^* - \mu/\tan \beta) & m_U^2 + m_i^2 \end{bmatrix} \]
Link to the Dark Matter in the Universe?

Parameters of the SUSY model \( \Rightarrow \) predictions for the relic density of dark matter

Interpretation in a simplified model

cMSSM (constrained Minimal Supersymmetric Standard Model)

Five parameters:
- \( m_0, m_{1/2} \) particle masses at the GUT scale
- \( A_0 \) common coupling term
- \( \tan \beta \) ratio of vacuum expectation value of the two Higgs doublets
- \( \mu \) (sign \( \mu \)) Higgs mass term

\[
\rho_x \sim m_x n_x, \quad n_x \sim \frac{1}{\sigma_{ann}(\chi \chi \rightarrow \ldots)}
\]

regions of parameter space which are consistent with the measured relic density of dark matter (WMAP,....)
9.4 Supersymmetry pre-LHC

The Search for

SUSY at LEP
9.4.1 Indirect SUSY searches

- Measure branching fractions of rare decays or search for forbidden decays.
  → Potentially enhanced by SUSY particles “in loops”:

- Measurement of $\mu$ anomalous magnetic moment ($g_\mu - 2$): Brookhaven
SUSY Dark Matter constraint

- mSUGRA parameter strongly constrained by cosmology ("blue bands")
- Annihilation and co-annihilation of dark-matter particles, etc.

mSUGRA $A_0=0$, $\tan(\beta)=10$, $\mu>0$

'Select point' region: significant $h$ component to LSP enhances annihilation to gauge bosons

'Focus point' region: $h$ component to LSP enhances annihilation to gauge bosons

'Select point' region: $h$ component to LSP enhances annihilation to gauge bosons

'Bulk' region: $t$-channel slepton exchange - LSP mostly Bino.

'Select point' region: $h$ component to LSP enhances annihilation to gauge bosons

'Select point' region: $h$ component to LSP enhances annihilation to gauge bosons

'Select point' region: $h$ component to LSP enhances annihilation to gauge bosons

Also 'rapid annihilation funnel' at Higgs pole at high $\tan(\beta)$,
Precision experiments and cosmology

- $b \rightarrow s\gamma$ excluded
- $g_\mu^{-2}$ favoured
- Dark matter favoured

cMSSM/mSUGRA already quite confined
Direct searches – general approach

- R-parity conserving Supersymmetry means **missing transverse momentum** in the final state

The simplest possible search strategy:
- A harsh cut on the missing transverse momentum will strongly reduce the amount expected events from known Standard Model Processes.
Direct searches – limit settings

- What if instead the number of observed events is consistent with the SM prediction?

For a given signal hypothesis, one can compute if the number of expected (observed) SM events differs from the expected (observed) SM + SUSY yield for more than 2 gaussian equivalent sigmas.

If yes, the model is excluded (either expected to be excluded or excluded)
Direct searches for sleptons at LEP

- Clear domain of LEP are Slepton & Chargino/Neutralino searches ($e^+e^-, E_{cm} \sim 200$ GeV)

- Excluded up to masses of 80 ... 100 GeV ($\sim E_{cm}/2$)

$\tilde{\ell} \rightarrow \ell \chi_1^0$ (2 leptons + missing E)

OPAL stau event candidate

\[ \sqrt{s} = 183-208 \text{ GeV} \]

ADLO Preliminary

\[ M_{\tilde{\tau}} < M_Z \]

Observed

Expected

Excluded at 95% CL

($\mu=200 \text{ GeV}/c^2$, $\cos\beta=1.5$)

\[ \Delta M \]

Small
Direct searches for charginos at LEP

Large $m_0$ ($\tilde{\ell}$ are heavy)

Small $m_0$ ($\tilde{\ell}$ are light)

Main SM backgrounds (WW, ZZ production) suppressed by requiring large missing mass or missing energy in the event.
The two classical SUSY signatures at the Tevatron

1. Search for Squarks and Gluinos: Jet + $E_T^{\text{miss}}$ signature produced via QCD processes

2. Search for Charginos and Neutralinos: Multilepton + $E_T^{\text{miss}}$ signature produced via electroweak processes (associated production – not discussed here)

\[
\tilde{\chi}_2 \tilde{\chi}_1^\pm \rightarrow l^\pm l^\pm l^\pm \tilde{\chi}_1^0 \tilde{\chi}_1^0 X
\]
Search for Squarks and Gluinos

- Three different analyses, depending on squark / gluinos mass relations:
  
  (i) dijet analysis
  small $m_0$, $m($squark$) < m($gluino$)$

  (ii) 3-jet analysis
  intermediate $m_0$ $m($squark$) \approx m($gluino$)$

  (iii) Gluino analysis
  large $m_0$, $m($squark$) > m($gluino$)$

- Main backgrounds: $Z \rightarrow \nu\nu +$ jets, $tt$, $W +$ jet production

- Event selection:
  * require at least 2, 3 or 4 jets with $P_T > 60 / 40 / 30 / 20$ GeV

  * veto on isolated electrons and muons

  * isolation of $E_T^{\text{miss}}$ and all jets

  * optimization of the final cuts $\rightarrow$ discriminating variables
Search for Squarks and Gluinos (cont.)

No excess above background from Standard Model processes

→ No evidence for SUSY (yet)  →  Set limits on masses of SUSY particles

**Expected background:**

<table>
<thead>
<tr>
<th>samples</th>
<th>2-jets</th>
<th>3-jets</th>
<th>4-jets</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD</td>
<td>4.37±2.01</td>
<td>13.34±4.67</td>
<td>15.26±7.60</td>
</tr>
<tr>
<td>top</td>
<td>1.35±1.22</td>
<td>7.56±3.85</td>
<td>22.14±7.29</td>
</tr>
<tr>
<td>Z→νν+jets</td>
<td>3.95±1.09</td>
<td>5.39±1.74</td>
<td>2.74±0.95</td>
</tr>
<tr>
<td>Z→ll+jets</td>
<td>0.09±0.04</td>
<td>0.16±0.11</td>
<td>0.14±0.08</td>
</tr>
<tr>
<td>W→νν+jets</td>
<td>6.08±2.15</td>
<td>10.69±3.84</td>
<td>7.68±2.85</td>
</tr>
<tr>
<td>WW/WZ/ZZ</td>
<td>0.21±0.19</td>
<td>0.35±0.17</td>
<td>0.49±0.34</td>
</tr>
<tr>
<td>tot SM</td>
<td>16±5</td>
<td>37±12</td>
<td>48±17</td>
</tr>
</tbody>
</table>

**Observed events in data:**

<table>
<thead>
<tr>
<th>Region</th>
<th>Observed data</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-jets</td>
<td>45</td>
</tr>
<tr>
<td>3-jets</td>
<td>38</td>
</tr>
<tr>
<td>2-jets</td>
<td>18</td>
</tr>
</tbody>
</table>
Excluded regions in the $m(\text{squark})$ vs. $m(\text{gluino})$ plane

**Exclusion limits**
(incl. systematic uncertainties)*:

$m(\text{gluino}) > 290 \text{ GeV/c}^2$
$m(\text{squark}) > 375 \text{ GeV/c}^2$

*) uncertainties from structure functions, change of renormalization and factorization scale $\mu$ by a factor of 2, NLO calculation, default choice: $\mu = m(\text{gluino}), m(\text{squark})$ or $\frac{1}{2}(m(\text{gluino})+m(\text{squark}))$ for $gg$, $qq$, $qg$ production
What do the LHC data say?
What processes are we looking for?

- ATLAS has set up dedicated search strategies for all production mechanism
- Only strong production (mainly 3rd generation) covered in this seminar
What we are typically doing

- **Heavy sparticles produced** in the primary collision
- They **decay into lighter objects**, emitting (high) $P_T$ jets and possibly other objects (leptons, photons) and MET (LSP)
- A “typical” SUSY event will have **large MET and large $H_T$**
- **Useful variables:**
  \[
  H_T = \sum_{jets} p_T^{jets} (+ \sum_l p_T^l + ...) \\
  M_{eff} = E_T^{miss} + H_T
  \]
- **But also other variables with well defined kinematical end point for the SM background**

- **$M_T$ (lepton-MET):** end point at $M_W$ if produced in $W$ decay
A complementary approach

Look for kinematic variables which have a well defined kinematical end-point for Standard Model processes

- An example: the transverse mass in events where a W boson is produced and decays into a lepton and a neutrino

- Beyond $m_W$, the Standard Model background decreases fast

- Think about $m_{T2}$ in events with 2 leptons:
  - It will have a kinematical end-point at $m_W$ for the SM background

$$m_{T2}(p_T^{\ell_1}, p_T^{\ell_2}, p_T^{\text{miss}}) = \min_{q_T \neq r_T = p_T^{\text{miss}}} \left\{ \max\left[ m_T(p_T^{\ell_1}, q_T), m_T(p_T^{\ell_2}, r_T) \right] \right\}$$
SUSY final states, there are many ….
<table>
<thead>
<tr>
<th>Process</th>
<th>Final States</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g \rightarrow b \bar{b} \chi_1^0 l$</td>
<td>$2\ell$ $2\nu$ $6j$ $H_T$</td>
</tr>
<tr>
<td>$g \rightarrow t \bar{b} \chi_0^+ l$</td>
<td>$g \rightarrow b \bar{b} \chi_1^0 q$</td>
</tr>
<tr>
<td>$g \rightarrow b \bar{b} \chi_1^0 \bar{l}$</td>
<td>$g \rightarrow t \bar{b} \chi_0^+ \bar{q_i}$</td>
</tr>
<tr>
<td>$g \rightarrow b \bar{b} \chi_0^+ q_i$</td>
<td>$g \rightarrow t \bar{b} \chi_1^0 \bar{q_i} q_k$</td>
</tr>
<tr>
<td>$g \rightarrow b \bar{b} \chi_1^0 \bar{l}$</td>
<td>$g \rightarrow t \bar{b} \chi_1^0 \bar{q}$</td>
</tr>
<tr>
<td>$g \rightarrow b \bar{b} \chi_1^0 \bar{q}$</td>
<td>$g \rightarrow t \bar{b} \chi_1^0 \bar{q}$ $q_k$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Process</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$g \rightarrow b \bar{b} \chi_1^0 l$</td>
<td>$2\ell$ $2\nu$ $6j$ $H_T$</td>
</tr>
<tr>
<td>$g \rightarrow t \bar{b} \chi_0^+ l$</td>
<td>$g \rightarrow b \bar{b} \chi_1^0 q$</td>
</tr>
<tr>
<td>$g \rightarrow b \bar{b} \chi_1^0 \bar{l}$</td>
<td>$g \rightarrow t \bar{b} \chi_0^+ \bar{q_i}$</td>
</tr>
<tr>
<td>$g \rightarrow b \bar{b} \chi_0^+ q_i$</td>
<td>$g \rightarrow t \bar{b} \chi_1^0 \bar{q_i} q_k$</td>
</tr>
<tr>
<td>$g \rightarrow b \bar{b} \chi_1^0 \bar{l}$</td>
<td>$g \rightarrow t \bar{b} \chi_1^0 \bar{q}$</td>
</tr>
<tr>
<td>$g \rightarrow b \bar{b} \chi_1^0 \bar{q}$</td>
<td>$g \rightarrow t \bar{b} \chi_1^0 \bar{q}$ $q_k$</td>
</tr>
</tbody>
</table>
Strong production

- Targeting generic strong production of gluinos and squarks.
- The exact decay chain depends on the SUSY mass hierarchy.

![Diagram of strong production]

- Two analyses drive the limit with 8 TeV data:

<table>
<thead>
<tr>
<th></th>
<th>0-lepton (ATLAS-CONF-2012-109)</th>
<th>1-lepton (ATLAS-CONF-2012-104)</th>
</tr>
</thead>
<tbody>
<tr>
<td>leptons</td>
<td>Veto any e or $\mu$ above 10 GeV</td>
<td>One isolated e or $\mu$ above 25 GeV</td>
</tr>
<tr>
<td>jets</td>
<td>2 to 6 jets with $p_T &gt; 60$ GeV (leading jet $p_T &gt; 130$ GeV)</td>
<td>4 jets with $p_T &gt; 80$ GeV</td>
</tr>
<tr>
<td>Other selections</td>
<td>MET &gt; 160 GeV, reject multijet with cuts on MET/$M_{\text{eff}}$, and angle between jets and MET</td>
<td>MET &gt; 250 GeV, $M_T &gt; 100$ GeV, additional selection on MET/$M_{\text{eff}}$</td>
</tr>
<tr>
<td>Final selection</td>
<td>$M_{\text{eff}}$</td>
<td>$M_{\text{eff}}$</td>
</tr>
<tr>
<td>Signal Region</td>
<td>≥ 2 jets</td>
<td>≥ 3 jets</td>
</tr>
<tr>
<td>---------------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>$E_T^{\text{miss}}$</td>
<td>&gt; 130</td>
<td>&gt; 130</td>
</tr>
<tr>
<td>Leading jet $p_T$</td>
<td>&gt; 130</td>
<td>&gt; 130</td>
</tr>
<tr>
<td>Second jet $p_T$</td>
<td>&gt; 40</td>
<td>&gt; 40</td>
</tr>
<tr>
<td>Third jet $p_T$</td>
<td>–</td>
<td>&gt; 40</td>
</tr>
<tr>
<td>Fourth jet $p_T$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\Delta \phi(\text{jet}, E_T^{\text{miss}})_{\text{min}}$</td>
<td>&gt; 0.4</td>
<td>&gt; 0.4</td>
</tr>
<tr>
<td>$E_T^{\text{miss}}/m_{\text{eff}}$</td>
<td>&gt; 0.3</td>
<td>&gt; 0.25</td>
</tr>
<tr>
<td>$m_{\text{eff}}$ [GeV]</td>
<td>&gt; 1000</td>
<td>&gt; 1000</td>
</tr>
</tbody>
</table>

$$m_{\text{eff}} = \sum_{i=1}^{n} |\vec{p}_T^{\text{jet } i}| + E_T^{\text{miss}}$$
Strong production

• **No excess** above SM in any of the signal regions:
  
  – interpreted first as a model-independent 95% C.L. limit on $\sigma_{\text{vis}}$ of BSM processes
  
  – then as an exclusion limit in specific SUSY models
CMS razor

If produced at threshold, then the jet and neutralino 4-vectors in the CM frame are

\[ p_{j_1} = \frac{M_\Delta}{2} (1, \hat{u}_1), \quad p_{j_2} = \frac{M_\Delta}{2} (1, \hat{u}_2), \]
\[ p_{\tilde{\chi}_1} = \frac{M_\Delta}{2} \left( \frac{2M_{\tilde{q}}}{M_\Delta} - 1, -\hat{u}_1 \right), \quad p_{\tilde{\chi}_2} = \frac{M_\Delta}{2} \left( \frac{2M_{\tilde{q}}}{M_\Delta} - 1, -\hat{u}_2 \right) \]

The CM frame is unknown (2 neutralinos), but it can be estimated as that that equals the magnitude of the jet 3-vectors. Then

\[ \beta_R \equiv \frac{E_{j_1} - E_{j_2}}{p_{j_1}^{Z_1} - p_{j_2}^{Z_2}} \]

Defining \( M_T^R \) and \( M_R \) as follow they both have an upper limit at \( M_\Delta \). The razor variable \( R \) is powerful in reducing the background (exponentially falling for background, peaking at 0.5 for signal)

\[ M_T^R \equiv \sqrt{\frac{E_T^{miss}(\vec{p}_{j_1}^{Z_1} + \vec{p}_{j_2}^{Z_2}) - E_T^{miss}(\vec{p}_{j_1}^{Z_1} + \vec{p}_{j_2}^{Z_2})}{2}} \]
\[ M_R \equiv 2|\vec{p}_{j_1}^R| = 2|\vec{p}_{j_2}^R| \]
\[ R \equiv \frac{M_T^R}{M_R} \]
CMS razor

CMS $\sqrt{s} = 7$ TeV
HAD box $\int L dt = 4.7$ fb$^{-1}$

- Data
- SM Total
- V+jets 1st
- $t\bar{t}$+jets 1st + effective 2nd

Events/(0.013)

$R^2$

CMS $\sqrt{s} = 7$ TeV
HAD box $\int L dt = 4.7$ fb$^{-1}$

- Data
- SM Total
- V+jets 1st
- $t\bar{t}$+jets 1st + effective 2nd

Events/(40 GeV)

$M_R$ [GeV]

$\tan(\beta) = 10$
$A_{\tau} = 0$ GeV
$\mu > 0$
$m_0 = 173.2$ GeV

Hybrid CLs 95% C.L. Limits

Razor Inclusive

- Median Expected Limit
- Expected Limit $\pm 1\sigma$
- Observed Limit
- Observed $\pm 1\sigma$ (theory)
- HAD Observed Limit
- Leptons Observed Limit

No EWSB

$\text{LEP2}\gamma^*$
direct $3^{rd}$ generation squark production

- The **stops/sbottoms** constrained by naturalness to be **not heavier** than $\sim 1$ TeV

- Sbottom decays:
  - $\tilde{b}_1 \rightarrow b \tilde{\chi}^0_1$
  - $\tilde{b}_1 \rightarrow t \tilde{\chi}^\pm_1$

- Stop decays:
  - $\tilde{t}_1 \rightarrow t \tilde{\chi}^0_1$
  - $\tilde{t}_1 \rightarrow b \tilde{\chi}^\pm_1$

---

\[ \sigma_{\text{tot}}(pb): pp \rightarrow \text{SUSY} \]
\[ \sqrt{S} = 8 \text{ TeV} \]
direct sbottom - 2 b-jets + $E_T^{\text{miss}}$

- $m_{CT}(bb)$: similar concept as for $m_{T2}$
- It has an end-point at $(m_{\text{prod}}^2 - m_{\text{inv}}^2)/m_{\text{prod}}$

- for large $\Delta m(b, \tilde{X}_1^0)$:
  - Look for 2 b-jets (veto on third jet), large $E_T^{\text{miss}}$
  - Use $M_{CT}$ to suppress top; Main background: $Z (\rightarrow \nu \nu) + b$-jets

- for small $\Delta m(b, \tilde{X}_1^0)$:
  - Small missing transverse momentum.
    - Focus on a sbottom pair recoiling against a jet
Direct sbottom search limits

- Plot sbottom mass on one axis, neutralino mass on the other axis:
  - The sbottom mass determines the cross section
  - The neutralino mass determines the kinematics
Direct stop search limits

\[ \tilde{t}_1 \rightarrow t\tilde{\chi}_1^0 \]

- Complex final state with many handles
- Analised in final states with 0/1/2 leptons
1-lepton stop (CMS)

Basic selection - 1 lepton, 4 jets and large MET.

Additional selections based on the hadronic top reconstruction on $M_T$ like quantities allows to suppress the ttbar background.
1-lepton stop (CMS)