1. Electromagnetic showers

The aim of the following exercise is to get a handle on the features of an electromagnetic shower. A very schematic model for the shower is the following. After each radiation length $X_0$, the number of particles is doubled (at the first stage, after one radiation length, an incoming electron emits a photon. After another radiation length, the electron will radiate a photon, while the photon produced in the first interaction creates an electron-positron pair, and so on...). Assume that at each stage, the energy is split equally among the particles emitted in one interaction. The multiplication process continues until the energy of the particles reaches the critical energy. Determine:

- The number of particles present in the shower as a function of the shower depth. [2 points]
- The dependence of the shower maximum (that is, the position at which the number of particles is maximum) on the incoming primary particle energy and the calorimeter critical energy. [2 points]

2. Hadronic showers

The aim of the exercise is to get a handle on why hadronic particles in non-compensating calorimeters have a response that depends in a non linear way on the incoming particle energy.

Assume that a hadronic shower is made up of two components: a pure hadronic component and an electromagnetic component. The pure electromagnetic component consists of mainly $\pi^0$'s produced in a hadronic decay that immediately decay through $\pi^0 \rightarrow \gamma\gamma$. Non-compensating calorimeters are characterised by the fact that the electromagnetic and hadronic component of the hadronic shower have a different response. This, combined with the fact that the relative importance of the electromagnetic and hadronic component of the shower is energy dependent, makes calorimeter response to hadrons non-linear.

Let's say that if the incoming pion energy is $E$, than a fraction $f_0$ of this will be seen in the form of photons from $\pi^0$ decays, and measured with a response 1. The remaining incoming energy will be measured with an efficiency $h < 1$.

- Write down an expression for the pion energy response, as a function of $E$, $f_0$, $h$. [2 points]
- Write down an expression for the so-called $e/\pi$ ratio, that is, the ratio of the energy measured in the calorimeter for an electron and a pion of the same incoming energy. [2 points]

We now want to get an insight on the $f_0$ dependency on the pion energy. We therefore assume that on each interaction, the available energy is equally split among two charged pions and one neutral pion, until a threshold $E_0$ for pion production is reached.

- Compute the dependency of $f_0$ on the incoming pion energy [2 points]. The following formulas might be helpful:

$$\sum_{0}^{n} x^k = \frac{1-x^{n+1}}{1-x}$$  (1)
\[
\alpha^{\log\beta} = \beta^{\log\alpha}
\]

- Make a sketch of the resulting $e/\pi$ ratio as a function of the incoming particle energy using ROOT. [2 points]

3. **Pythia 8** Download and install Pythia 8 (available at http://home.thep.lu.se/~torbjorn/Pythia.html). Compile it and execute the examples in the main directory. Attach the log of the example number 1. [1 point]