

Problem Set for Hadron Collider Physics 2015
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Problem Set 5

Your solutions have to be handed in by 10:10 am on Tuesday, June 16th 2015.
Please drop them into the mailbox number 1 on the ground floor of the
Gustav-Mie building!

1. **Weak interactions**

- (a) Show that the projection operator $P_L := \frac{1}{2}(1 - \gamma^5)$, which projects out the left-chiral state of a fermion, obeys the relation $P_L = P_L^2$. [1 point]
- (b) Show that the $(V - A)$ expression for the charged weak current can be rewritten as a pure vectorial current involving the left-chiral components of the involved fermions. That is, show the equivalence of the two expressions below (focusing in the notation on a $W e \nu$ vertex). [2 points]

$$\bar{\nu} \gamma_\mu \frac{1}{2} (1 - \gamma_5) u_e = \bar{\nu}_L \gamma_\mu e_L \quad (1)$$

- (c) In a similar way, show that the expression for the weak neutral current can be rewritten as a current involving left-chiral fermion components plus a current involving right-chiral fermion components. That is, show that: [2 points]

$$\bar{e} \gamma_\mu \frac{1}{2} (C_V - C_A \gamma_5) e = \bar{e}_L g_L \gamma_\mu e_L + \bar{e}_R g_R \gamma_\mu e_R \quad (2)$$

2. **Electroweak theory**

The electroweak current can be written in the combined electroweak theory as

$$-ig \vec{j}_\mu \cdot \vec{W}^\mu - i \frac{g'}{2} j_\mu^Y B^\mu, \quad (3)$$

where \vec{j}_μ is the weak isospin current, j_μ^Y is the hypercharge current, and \vec{W}^μ and B^μ are the gauge fields.

- (a) Write down the expression for the four physical fields W^\pm , A , and Z in terms of the quantities appearing on the expression above and the weak mixing angle θ_W [2 points].
- (b) Given the relationship between the electromagnetic current, \vec{j}_μ , and j_μ^Y , what is the relation between the electric charge e and g, g' ? [2 points]
- (c) From the neutral current interaction term, derive an expression for the vectorial and axial couplings C_V, C_A to fermions in terms of the hypercharge and weak isospin of the fermions. [2 points]