1. Scintillator and Photomultiplier Efficiencies
   The quantum efficiency of a photomultiplier tube is 18%.
   (a) For 8 photons hitting the photocathode, what is the probability of emitting
       • no electrons
       • at least one electron
       • exactly one electron
   (b) The photomultiplier is used to detect light from an organic scintillator with a
density of 1.15 g cm$^{-3}$ and $4 \times 10^3$ photons being emitted per MeV of particle
   energy loss. The probability of scintillation photons reaching the photomultiplier
   is only 5%, due to losses in the scintillator and light guide. How thick a scintillator
   slab is required to detect minimum ionizing particles with an efficiency of at least
   99%?

2. Electromagnetic Shower
   A homogeneous calorimeter is used to measure the energy of electrons or photons. The
   electromagnetic shower started by the incident particle can be modelled as a shower
   of electrons/positrons and photons using the simple assumptions:
   • Each electron/positron with an energy greater than the a critical value $E_c$ gives
     up half of its energy to a bremsstrahlung photon when traveling one radiation
     length $X_0$. An electron/positron with $E < E_c$ ceases to radiate and loses all its
     energy locally by ionization.
   • Each photon travels one radiation length and then undergoes pair production. It
     should be assumed that the electron and positron each take half of the photon’s
     energy.
   (a) How many particles are there after $t$ radiation lengths? What is the energy per
   particle at that point, for an initial particle energy $E_0$?
(b) Show that the maximum number of particles can be found at

\[ t_{\text{max}} = \frac{\ln E_0/E_c}{\ln 2} \]

(c) What is the total track length of charged particles in the shower up to \( t_{\text{max}} \)?

(3 Points)

3. NUMERICAL SIMULATION - Scintillator Rod

Consider a plastic scintillator (density 1.032 g cm\(^{-3}\)) with cylindrical shape, the radius being 1 cm and the length 1 m. The scintillator has a light attenuation range of 1.6 m, a refractive index of 1.58 and is surrounded by air. Scintillation photons are collected by a photocathode at one end of the rod. Minimum ionizing particles cross the scintillator perpendicular to and through the cylinder axis, emitting one photon every 100 eV of deposited energy. Photons are emitted isotropically, and for simplicity consider all photons to be emitted on the cylinder axis.

(a) At first consider any photon impacting the side of the cylinder and not undergoing total internal reflection to be lost, and in addition attenuation losses in the scintillator material. Simulate the propagation of photons and determine the mean fraction of photons collected depending on the initial particle crossing point along the cylinder axis.

(b) Compare to a setup where a high reflective coating is put on the end of the rod opposite the photocathode, with a reflectivity > 99.99%.

(c) Compare a setup where the whole scintillator (except for the photocathode end) was coated with a thin Ag-Al layer (for simplicity assume a refractive index of \( \sim 1.3 \) and reflectivity of \( \sim 95\% \) for all angles smaller than the critical angle).

(4 Points)