Problem Set for Hadron Collider Physics 2015

Prof. Dr. Karl Jakobs, Dr. Karsten Köneke

Problem Set 1

Your solutions have to be handed in by 10:10 am on Tuesday, the 28th of April 2015. Please drop them into the mailbox number 1 on the ground floor of the Gustav-Mie building!

---

1. **Luminosity**

   The instantaneous luminosity for a circular particle accelerator can be expressed as

   $$\mathcal{L} = f N_b \frac{N_1 N_2}{4\pi \sigma_x \sigma_y},$$

   where f is the frequency of the bunch traversing the circular orbit, N_b is the number of particle bunches in the accelerator, N_1 and N_2 the number of particles per bunch, and σ_x and σ_y the transverse size of the particle beam spot at the interaction point. If the particle masses are much lighter than the beam energy, then one can make the approximation that the particles travel at the speed of light.

   At the LHC, 2808 proton bunches with 1.1×10^{11} protons per bunch are collided (at 8 TeV in 2012, and planned at 13 TeV in 2015). The beams are focused to sizes of $\sigma_x = \sigma_y = 20\,\mu$m at the interaction point.

   (a) What is the instantaneous luminosity at the LHC in units of $\mathrm{s}^{-1}\mathrm{cm}^{-2}$?

   (b) Assume that the LHC running time is 10^7 s each year, what is the integrated luminosity per year expressed in units of fb^{-1}?

   (c) The cross-section for Higgs-boson production is $\sigma_H = 51\,\mathrm{fb}$ at a center-of-mass energy of 13 TeV. The decay branching fraction of $H \to \gamma\gamma$ is 2.3×10^{-3}. How many $H \to \gamma\gamma$ events are produced at the LHC per year at 13 TeV, using the luminosities calculated in (a) and (b)?

   [2 points]

2. **Synchrotron Radiation**

   From electrodynamics, we know that accelerated charged particles radiate energy. The energy that is lost by a charged particle traveling in a circular orbit with radius R through synchrotron radiation, is given according to the equation (in natural units):

   $$\delta E = \frac{4\pi\alpha}{3} \frac{\gamma^4}{R},$$

   where $\alpha = \frac{e^2}{\hbar c}$ is the fine structure constant, and $\gamma = 1/\sqrt{1-v^2}$.

   (a) Show that the energy loss expressed in units of GeV is given by:

   $$\delta E[\mathrm{GeV}] = \frac{6.03 \times 10^{-18}}{R[\mathrm{m}]} \left(\frac{E}{m}\right)^4.$$

   (here the radius of the circular orbit is expressed in units of meters)

(b) For proton-proton collisions at the LHC (assume the design beam energy of 7 TeV), what is the energy loss for the protons due to synchrotron radiation per orbit?

(c) A possible future electron-positron collider to make precision measurements is being considered that could operate at a center-of-mass energy of 500 GeV. Why is a linear collider being considered instead of a synchrotron?

[3 points]

3. **Hadron Collision Kinematics**

At the LHC, pp collisions at a center-of-mass energy of 8 TeV take place. Assume that partons with momentum fractions of x_1 and x_2 collide in a hard interaction, where x_1 is the momentum fraction (Bjorken x) of the parton traveling in the positive z direction, and x_2 is the momentum fraction of the parton traveling in the $-z$ direction.

Here, one can assume that the partons are massless, and recall that the beam axis corresponds to the z-axis. (The collision occurs at the origin, the y-axis points upwards away from the earth's center, while the x-axis points towards the center of the LHC ring).

In the resulting reactions, the partons annihilate into two other massless particles, and in the center-of-mass reference frame of the two incoming partons, the outgoing particles are emitted transverse (perpendicular) to the beam axis.

(a) What is the center-of-mass energy in the parton-parton rest frame (expressed as a function of x_1 and x_2)?

(b) What are the transverse momenta (the momentum-component perpendicular to the beam axis) of the outgoing particles (expressed as a function of x_1 and x_2)?

(c) Calculate the pseudorapidity $\eta = -\ln(\tan\frac{\theta}{2})$ of the outgoing particles (in terms of x_1 and x_2) in the laboratory frame.
Hint: Recall that θ is the polar angle of the spherical coordinate system, with $\theta = 0$ corresponding to the positive z-axis direction.

(d) Assume $x_1 = 0.3$ and $x_2 = 0.5$. What is the pseudorapidity of the outgoing particles in the laboratory frame?
Hint: What is the valid range for θ?

[4 points]

4. **Stopping particles**

A muon behaves in matter to a good approximation as a minimum ionizing particle (MIP), that is, as a particle that is in the minimum of the $\frac{dE}{dx}$ distribution of the Bethe-Bloch formula. A typical MIP energy loss is 1.5 MeV/(g cm^{-2}), where this value is already multiplied by the density of the traversed material (which is very commonly done for many given values and curves, i.e., often $\frac{dE}{dx}\rho$ is given instead of $\frac{dE}{dx}$).

- Compute the thickness of a lead layer needed to stop a 1 GeV muon (typical cosmic ray energy) assuming that the muon behaves like a MIP over all its path.
- Given your knowledge of the Bethe-Bloch formula, do you think that the estimate just done is an overestimation or an underestimation of the actual thickness of lead needed? Why? [HINT: as the muon slows down, it's energy loss per unit length...]

[3 points]