2. Interaction of Charged Particles with Matter

2.1 Energy loss by ionisation and excitation
(Bethe-Bloch formula)

2.2 Bremsstrahlung

2.3 Multiple Coulomb Scattering

2.4 Range of particles in matter

2.5 Cherenkov Radiation

2.6 Transition Radiation

2.7 Summary

Interaction of Charged Particles with Matter

- Detection through interaction of the particles with matter, e.g. via energy loss in a medium (ionization and excitation)

- Energy loss must be detected, made visible, mainly in form of electric signals or light

- Fundamental interaction for charged particles: electromagnetic interaction
  Energy is mainly lost due to interaction of the particles with the electrons of the atoms of the medium

  Cross sections are large: $\sigma \sim 10^{-17} - 10^{-16}$ cm^2 !!

  Small energy loss per collision, however, large number of collisions in dense materials

- Interaction processes (energy loss, scattering,..) are a nuisance for precise measurements and limit their accuracy

Overview on energy loss / detection processes

Charged particles	Photons, γ
Ionisation and excitation Bremsstrahlung	Photoelectric effect Compton scattering Pair creation
Cherenkov radiation Transition radiation	

2.1 Energy loss by ionisation and excitation

- A charged particle with mass m_0 interacts primarily with the electrons (mass m_e) of the atom;
  Inelastic collisions → energy loss

- Maximal transferable kinetic energy is given by: $T^{max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e / m_0 + (m_e / m_0)^2}$

  max. values: Muon with E = 1.06 GeV (γ = 10): E_{kin}^{max} ~100 MeV

  Two types of collisions:
  Soft collision: only excitation of the atom
  Hard collision: ionisation of the atom
  In some of the hard collisions the atomic electron acquires such a high energy that it causes secondary ionisation (δ-electrons).

  → Ionisation of atoms along the track / path of the particle;
  In general, small energy loss per collision, but many collisions in dense materials → energy loss distribution
  one can work with average energy loss

- Elastic collisions from nuclei cause very little energy loss, they are the main cause for deflection / scattering under large angles

History of Energy Loss Calculations: dE/dx

1915: Niels Bohr, classical formula, Nobel prize 1922.
1930: Non-relativistic formula found by Hans Bethe
1932: Relativistic formula by Hans Bethe

Bethe's calculation is leading order in pertubation theory, thus only z^2 terms are included.

Additional corrections:

- z^3 corrections calculated by Barkas-Andersen

- z^4 correction calculated by Felix Bloch
  (Nobel prize 1952, for nuclear magnetic resonance).
  Although the formula is called Bethe-Bloch formula the z^4 term is usually not included.

- Shell corrections: atomic electrons are not stationary

- Density corrections: by Enrico Fermi
  (Nobel prize 1938, for the discovery of nuclear reaction induced by slow neutrons).

Hans Bethe (1906-2005)

Studied physics in Frankfurt and Munich, emigrated to US in 1933. Professor at Cornell U.,

Nobel prize 1967 for the theory of nuclear processes in stars.

Bethe-Bloch Formula

Bethe-Bloch formula gives the mean energy loss (stopping power) for a heavy charged particle ($m_0 >> m_e$)*

$$-\frac{dE}{dx} = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

PDG 2008

A : atomic mass of absorber

$\frac{K}{A} = 4\pi N_A r_e^2 m_e c^2 / A = 0.307075 \text{ MeV g}^{-1}\text{cm}^2, \text{ for A} = 1\text{g mol}^{-1}$

z: atomic number of incident particle

Z: atomic number of absorber

I : characteristic ionization constant, material dependent

T^{max}: max. energy transfer (see previous slide)

δ ($\beta\gamma$): density effect correction to ionization energy loss

$x = \rho$ s , surface density or mass thickness, with unit g/cm^2, where s is the path length of the particle

dE/dx has the units MeV cm^2/g

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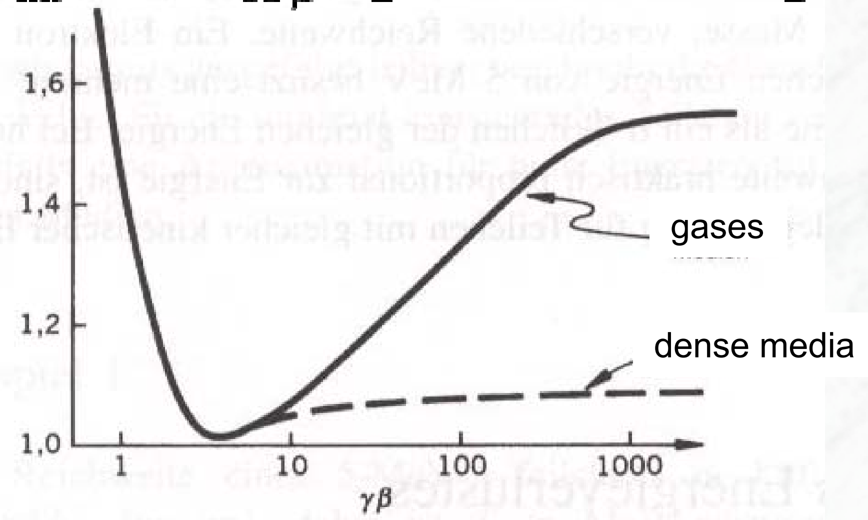
* note: Bethe-Bloch formula is not valid for electrons (equal mass, identical particles)

$$-\frac{dE}{dx} = K z^2 \frac{Z}{A} \frac{1}{\beta^2}\left[\frac{1}{2}\ln f(\beta) - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

Important features / dependencies:



- Energy loss is independent of the mass of the incoming particle
  → universal curve

- depends quadratically on the charge and velocity of the particle: ~ z^2/β^2

- dE/dx is relatively independent of the absorber
  (enters only via Z/A, which is constant over a large range of materials)

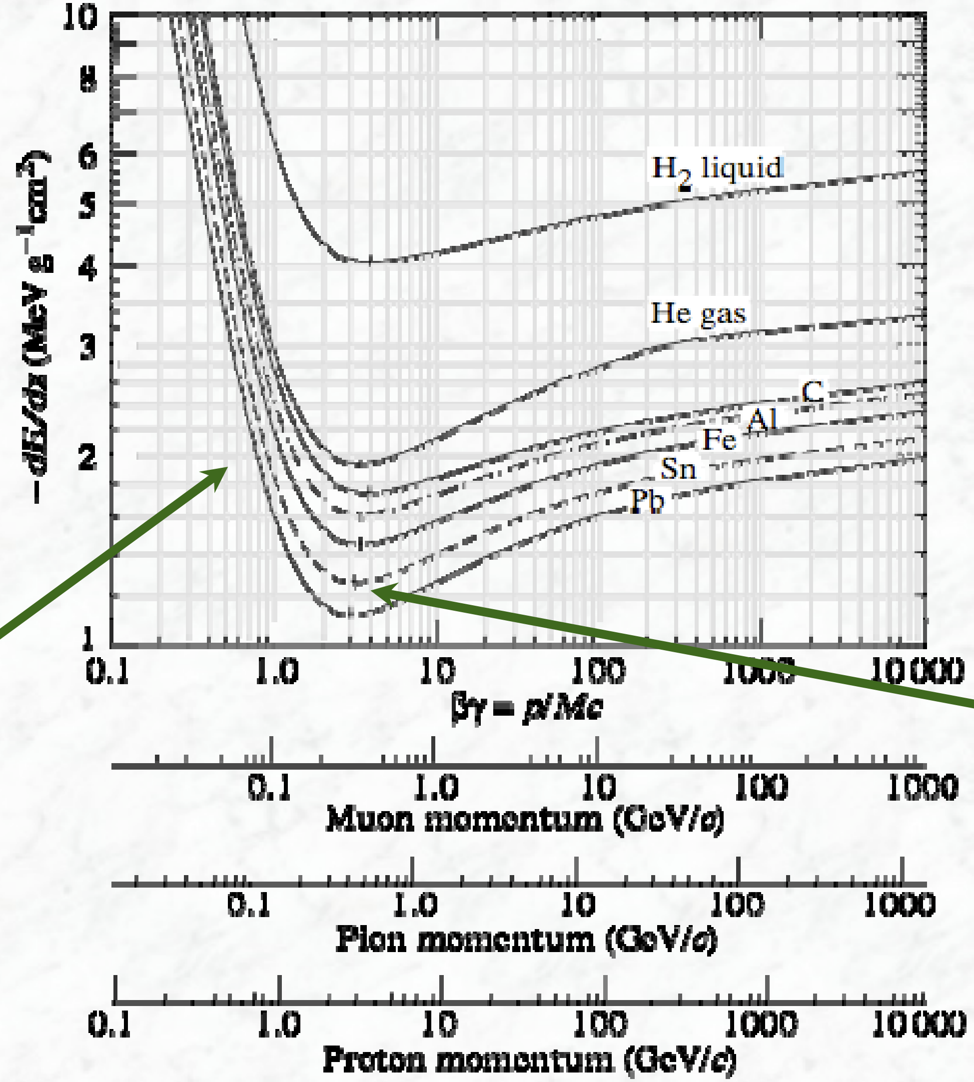
- Minimum for $\beta\gamma \approx 3.5$
  energy loss in the mimimum:

$$\left.\frac{dE}{dx}\right|_{\min} \approx 1.5 \frac{MeV \cdot cm^2}{g}$$

  (particles that undergo minimal energy loss are called "mimimum ionizing particle" = mip)

- Logarithmic rise for large values of $\beta\gamma$ due to relativistic effects is damped in dense media $\delta(\beta\gamma)$

Examples of Mean Energy Loss



1/β²

Figure 27.3: Mean energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, iron, tin, and lead. Radiative effects, relevant for muons and pions, are not included. These become significant for muons in iron for $\beta\gamma \gtrsim 1000$, and at lower momenta for muons in higher-Z absorbers. See Fig. 27.21.

PDG 2008

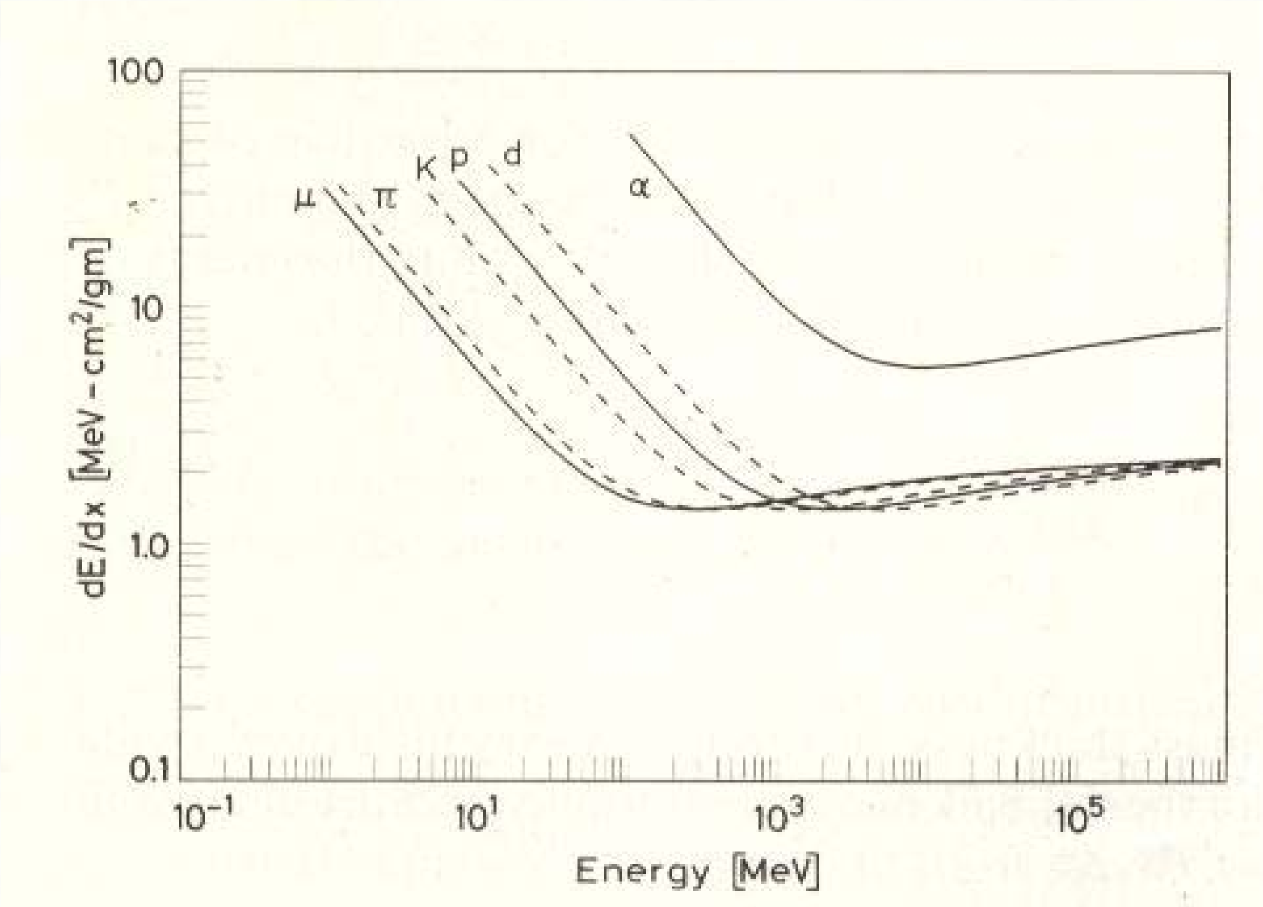
Bethe-Bloch formula:

$$-\frac{dE}{dx} = Kz^2 \frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln f(\beta) - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

Except in hydrogen, particles of the same velocity have similar energy loss in different materials.

The minimum in ionisation occurs at βγ = 3.5 to 3.0, as Z goes from 7 to 100

Consequence: dE/dx measurements can be used to identify particles

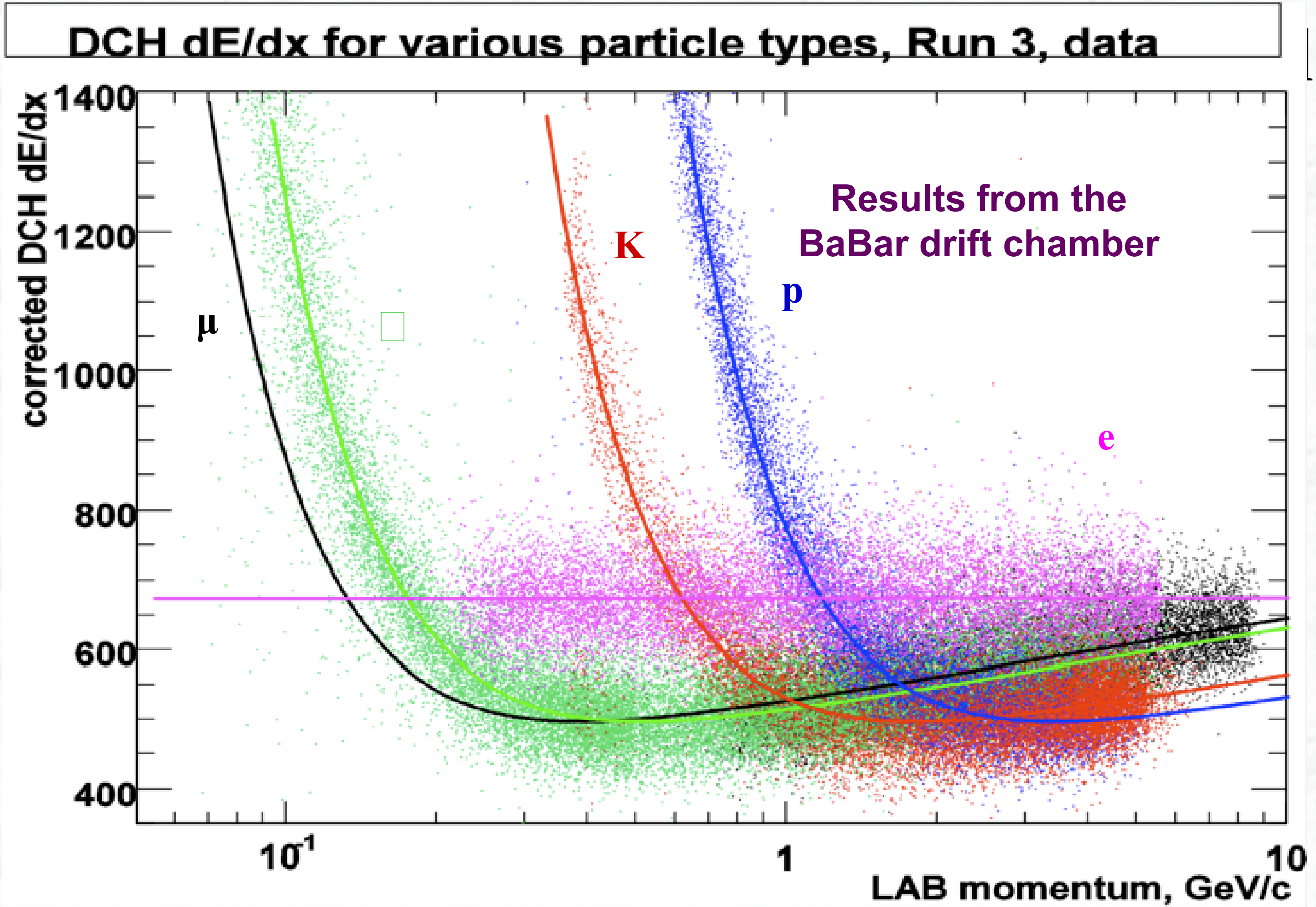


$$\beta\gamma = \frac{p}{E}\frac{E}{m} = \frac{p}{m}$$

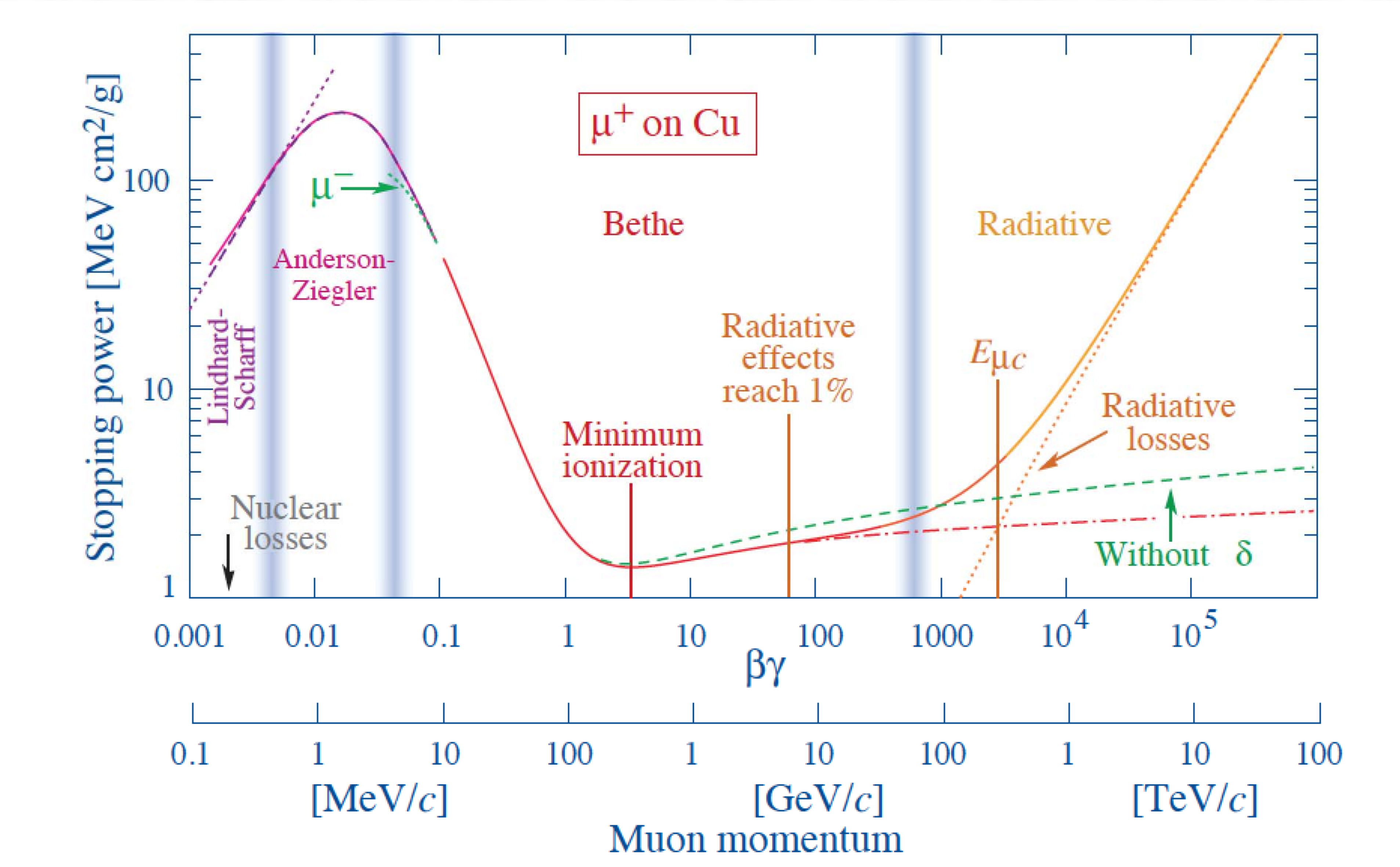
- universal curve as function of $\beta\gamma$ splits up for different particle masses, if taken as function of energy or momentum

→ a simultaneous measurement of dE/dx and p,E → particle ID

Example: BaBar experiment at SLAC



A simultaneous measurement of dE/dx and momentum p can provide particle identification. Works well in the low momentum range (< ~1 GeV)



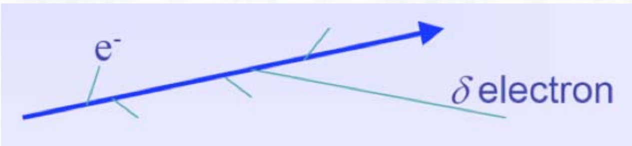
Mean energy loss –dE/dx for positive muons in copper as a function of $\beta\gamma = p/m_0c$ over nine orders of Magnitude in momentum (Particle Data Group, 2013).

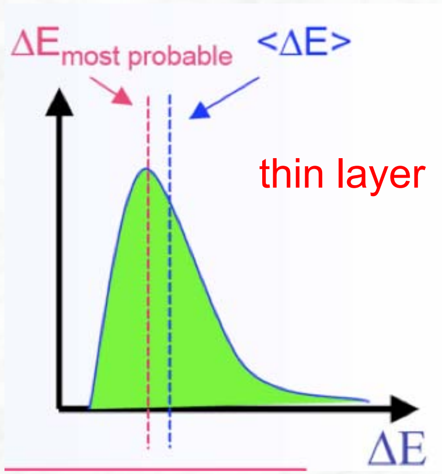
Fluctuations in Energy Loss

- A real detector (limited granularity) cannot measure $\langle dE/dx \rangle$;
- It measures the energy ΔE deposited in layers of finite thickness Δx;

- Repeated measurements → sampling from an energy loss distribution

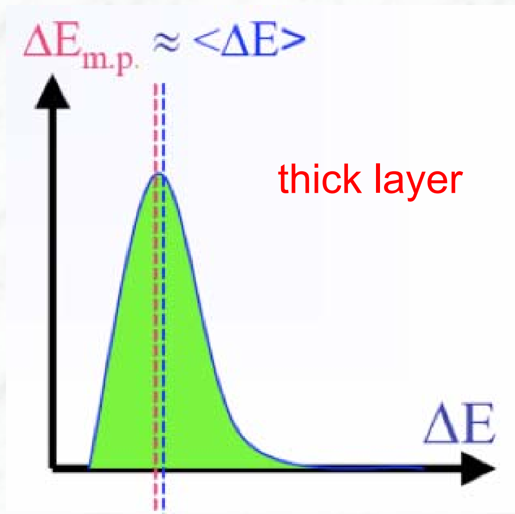
- For thin layers or low density materials, the energy loss distribution shows large fluctuations towards high losses, so called Landau tails.

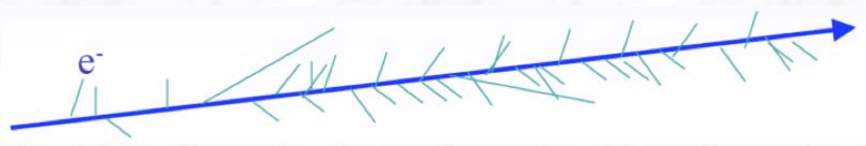




Example: Silicon sensor, 300 μm thick,
ΔE_{mip} ~82 keV, $\langle \Delta E \rangle$ ~115 keV

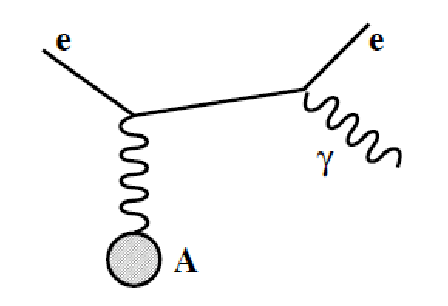
- For thick layers and high density materials, the energy loss distribution shows a more Gaussian-like distribution (many collisions, Central limit theorem)





2.2 Energy loss due to bremsstrahlung

- High energy charged particles undergo an additional energy loss (in addition to ionization energy loss) due to bremsstrahlung, i.e. radiation of photons, in the Coulomb field of the atomic nuclei



$$-\frac{dE}{dx}\bigg|_{Brems} = 4\alpha N_A \left(\frac{e^2}{mc^2}\right)^2 \ln\frac{183}{Z^{1/3}}\frac{Z(Z+1)}{A} Q^2 E$$

where: Q, m = electric charge and mass of the particle,
α = fine structure constant
A,Z = atomic number, number of protons of the material
N_A= Avogardo's number

- One can introduce the so-called radiation length X_0 defined via the above equation:

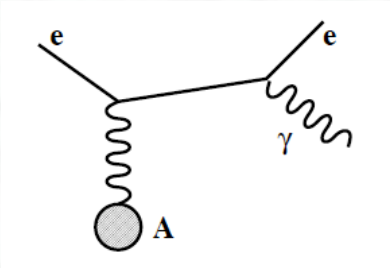
$$\frac{1}{X_0} = 4\alpha N_A \left(\frac{e^2}{m_e c^2}\right)^2 \ln\frac{183}{Z^{1/3}}\frac{Z(Z+1)}{A} \xrightarrow{\text{for electrons}} -\frac{dE}{dx}\bigg|_{Brems} := \frac{1}{X_0} E$$

note that in the definition of X_0 the electron mass is used (electron as incoming particle). It only depends on electron and material constants and chacterises the radiation of electrons in matter

Radiation lengths of various materials:

Material	Z	A	$X_0[g/cm^2]$	X_0/ϱ [cm]	$E_c[MeV]$
Wasserstoff	1	1.01	63	700000	350
Helium	2	4.00	94	530000	250
Lithium	3	6.94	83	156	180
Kohlenstoff	6	12.01	43	18.8	90
Stickstoff	7	14.01	38	30500	85
Sauerstoff	8	16.00	34	24000	75
Aluminium	13	26.98	24	8.9	40
Silizium	14	28.09	22	9.4	39
Eisen	26	55.85	13.9	1.76	20.7
Kupfer	29	63.55	12.9	1.43	18.8
Silber	47	109.9	9.3	0.89	11.9
Wolfram	74	183.9	6.8	0.35	8.0
Blei	82	207.2	6.4	0.56	7.40
Luft	7.3	14.4	37	30000	84
SiO_2	11.2	21.7	27	12	57
Wasser	7.5	14.2	36	36	83

$$\left.-\frac{dE}{dx}\right|_{Brems} = 4\alpha N_A \left(\frac{e^2}{mc^2}\right)^2 \ln\frac{183}{Z^{1/3}} \frac{Z(Z+1)}{A} Q^2 E$$



Most important dependencies:

- Material dependence $\frac{dE}{dx} \sim \frac{Z(Z+1)}{A}$

- Mass of the incoming particle: $\frac{dE}{dx} \sim \frac{1}{m^2}$

(light particles radiate more)

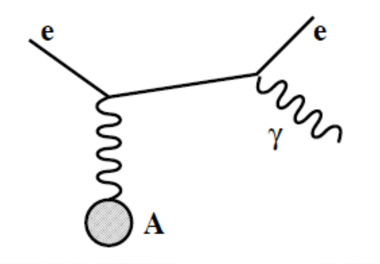
This is the reason for the strong difference in bremsstrahlung energy loss between electrons and muons

$$\left(\frac{dE}{dx}\right)_\mu \Big/ \left(\frac{dE}{dx}\right)_e \sim \frac{1}{40.000}$$

- Energy of the incoming particle $\frac{dE}{dx} \sim E$

This implies that this energy loss contribution will become significant for high energy muons as well

For electrons the energy loss equation reduces to



$$-\left.\frac{dE}{dx}\right|_{Brems} := \frac{1}{X_0}E \quad \Rightarrow \quad E(x) = E_0 e^{-x/X_0}$$

- The energy of the particle decreases exponentially as a function of the thickness x of the traversed material, due to bremsstrahlung;

After $x=X_0$: $$E(X_0) = \frac{E_0}{e} = 0.37E_0$$

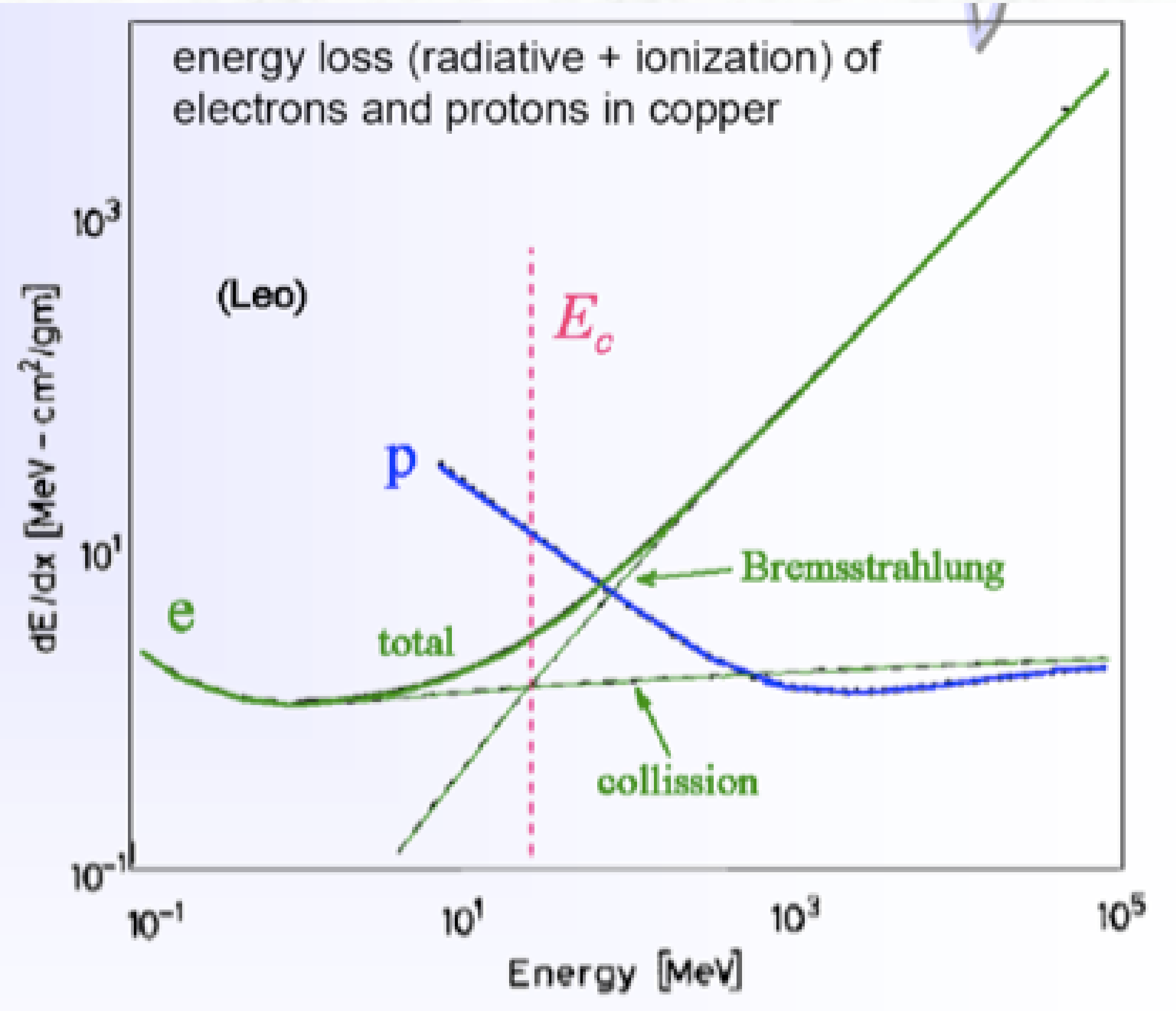
- Continuous 1/E energy loss spectrum, mainly soft radiation, with hard tail
- One defines the critical energy, as the energy where the energy loss due to ionization and bremsstrahlung are equal

$$-\left.\frac{dE}{dx}\right|_{ion}(E_c) = -\left.\frac{dE}{dx}\right|_{brems}(E_c)$$

useful approximations for electrons: (heavy elements)

$$E_c = \frac{550\,\mathrm{MeV}}{Z}$$

$$X_0 = 180\frac{A}{Z^2}\left(\frac{\mathrm{g}}{\mathrm{cm}^2}\right)$$

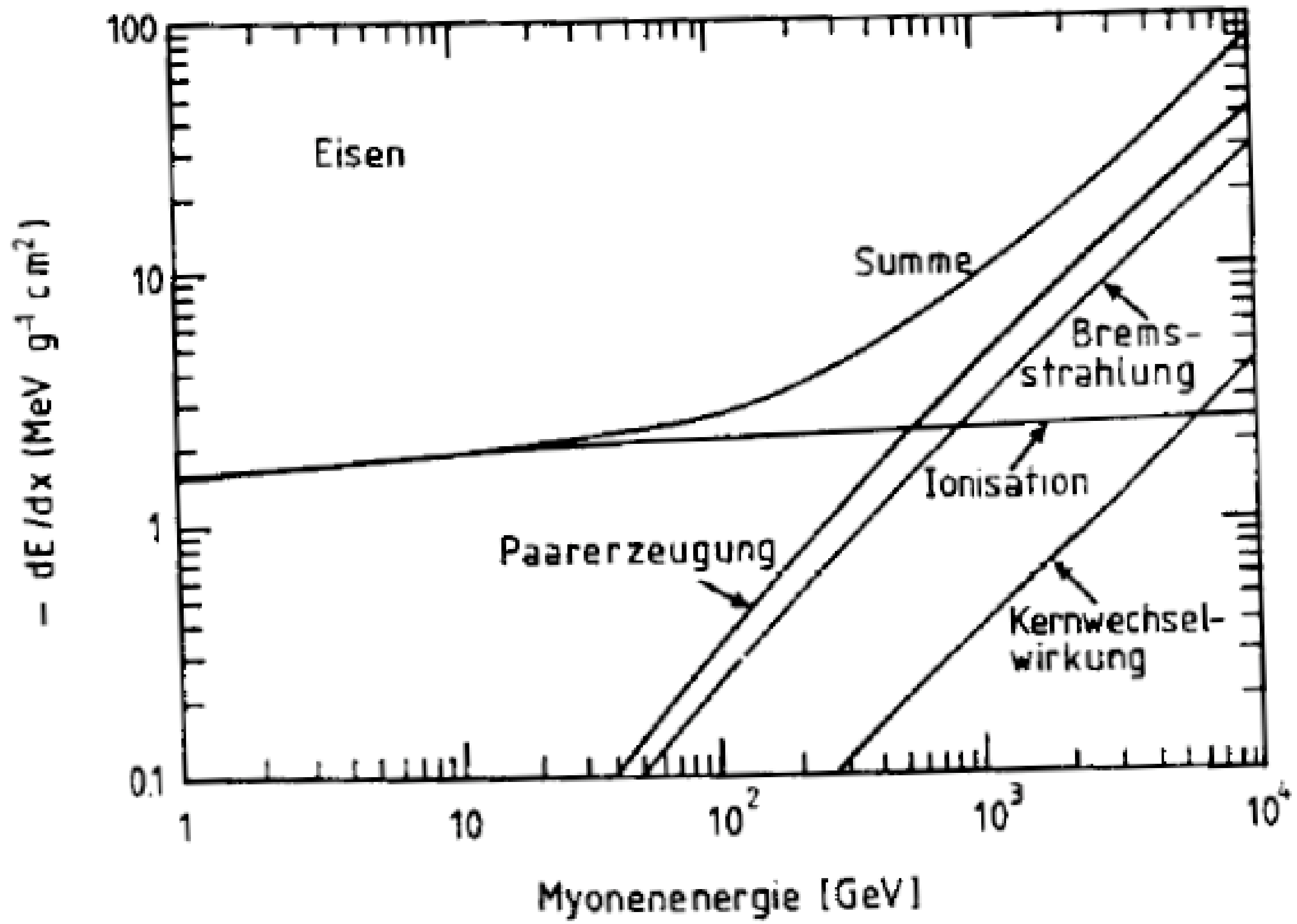


Critical energies in copper (Z = 29):

E_c (e) ≈ 19 MeV

$E_c(\mu)$ ≈ 1 TeV

- Muons with energies > ~10 GeV are able to penetrate thick layers of matter, e.g. calorimeters;
- This is the key signature for muon identification



Energy loss dE/dx for muons in iron;

- critical energy ≈ 870 GeV;
- At high energies also the pair creation μ (A) → $\mu\ e^+e^-$ (A) becomes important

Material	Z	X_0 (cm)	E_c (MeV)
H_2 Gas He	1 2	700000 530000	350 250
Li C	3 6	156 18.8	180 90
Fe Cu W Pb	26 29 74 82	1.76 1.43 0.35 0.56	20.7 18.8 8.0 7.4
Air SiO_2 Water	7.3 11.2 7.5	30000 12 36	84 57 83

Radiations lengths and critical energies for various materials (from Ref. [Grupen])

2.3 Multiple Coulomb Scattering

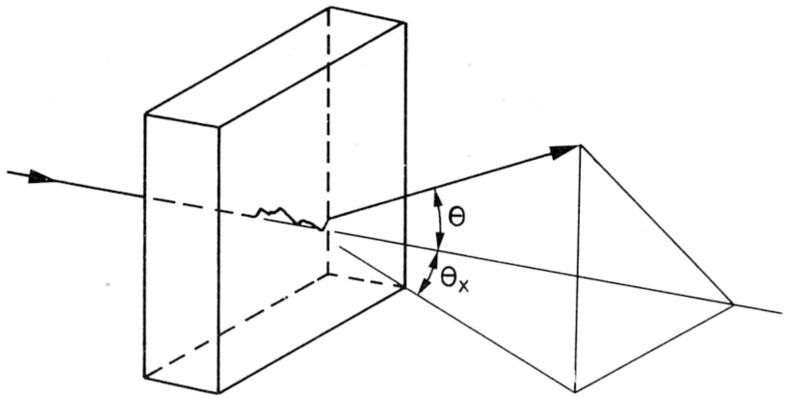
- So far: inelastic collisions with the atomic electrons

- In addition: charged particles passing through matter suffer repeated elastic Coulomb scattering from nuclei

  smaller cross-section / probability,
  basic process described by Rutherford formula: $\sim \; z^2 \cdot Z^2 \frac{1}{\sin^4 \theta/2}$

  → most of the collision result in small angular deflections
  ($m_{nucleus} >> m_{particle}$) → small energy transfer,
  however a large number of collisions or deflections

- Cumulative effect of small angle scatterings is smearing of direction
  Problem can be treated statistically;
  Probability density for deflection angle as function of thickness of material



- Small angle approximation ($\theta <$ ~10^o) (Molière theory):

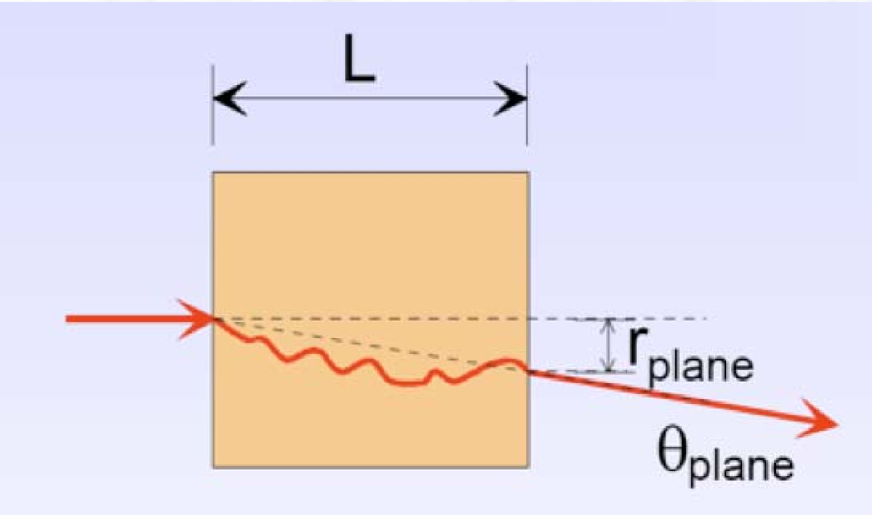
→ Gaussian probability density

mean: $\mu_\theta = 0$

Standard deviation:

$$\sigma_\theta^{plane} = \frac{13.6\,\text{MeV}}{\beta cp} \cdot z\sqrt{\frac{L}{X_0}}$$

$$\sigma_\theta = \sqrt{2} \cdot \sigma_\theta^x$$



where: X_0 = radiation length of the material

p = particle momentum (MeV/c)

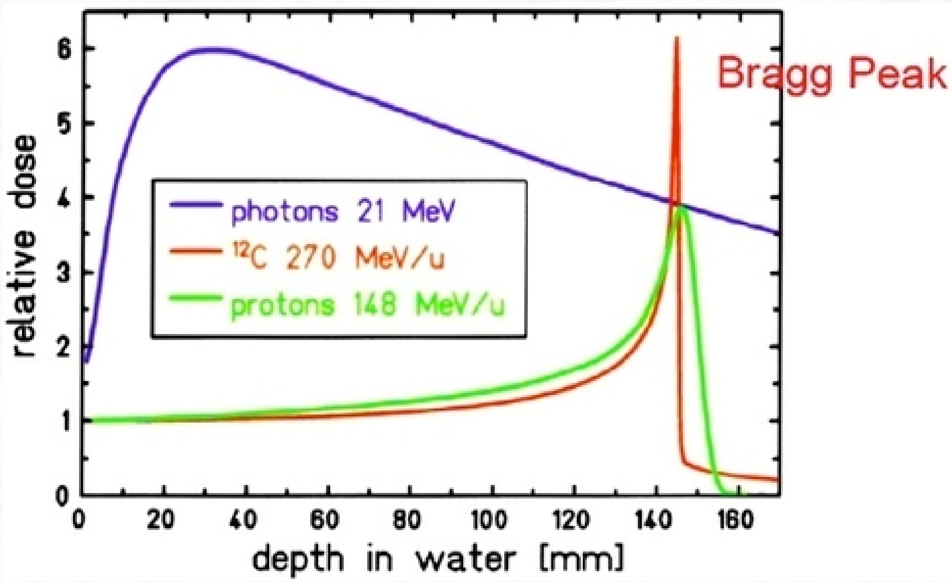
L = Length traversed in the medium

$\sigma_\theta^x = \sigma_\theta^{plane}$

2.4 Range of Charged Particles in Matter

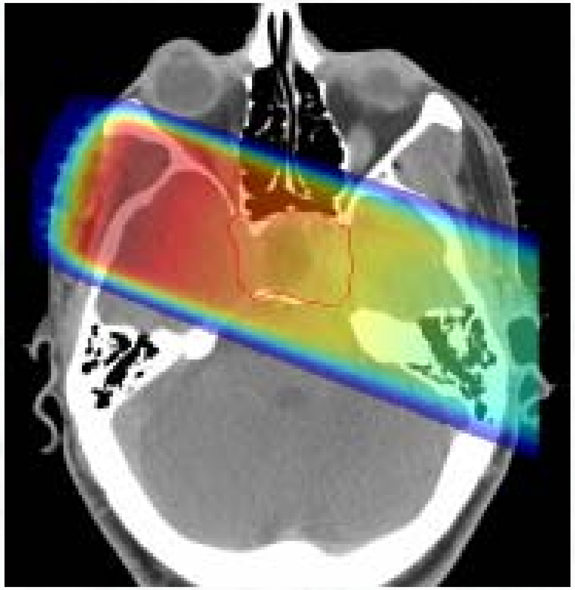
- Due to interaction processes particles loose energy in matter and will eventually be stopped.
- Energy loss via dE/dx (ionisation loss) depends on velocity (b g) and thereby for a given particle it depends on the penetration length in the medium

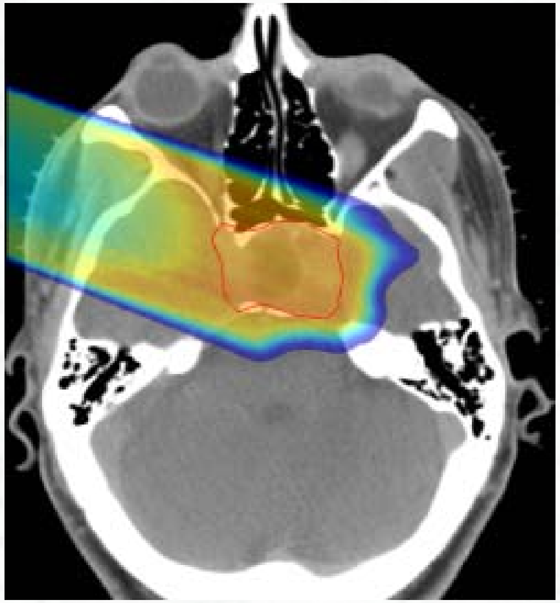
→ Bragg curve (energy loss –or radiation dose via deposited energy- as a function of path length (depth x) in the medium



- Application in medical treatment, hadron therapy

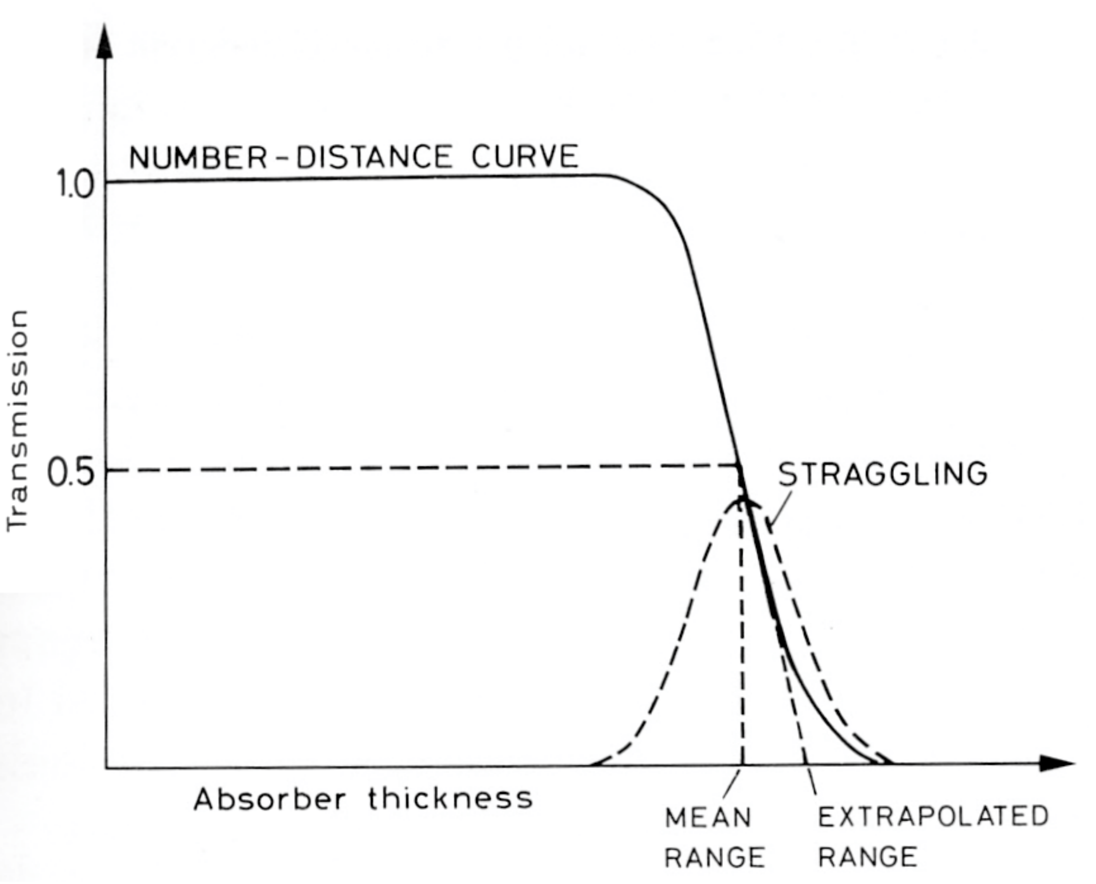
  (GSI Darmstadt, Univ. Heidelberg)





- Range can be measured using a transmission curve:

Transmission: = (# particles measured after absorber thickness x) / (# incident particles)



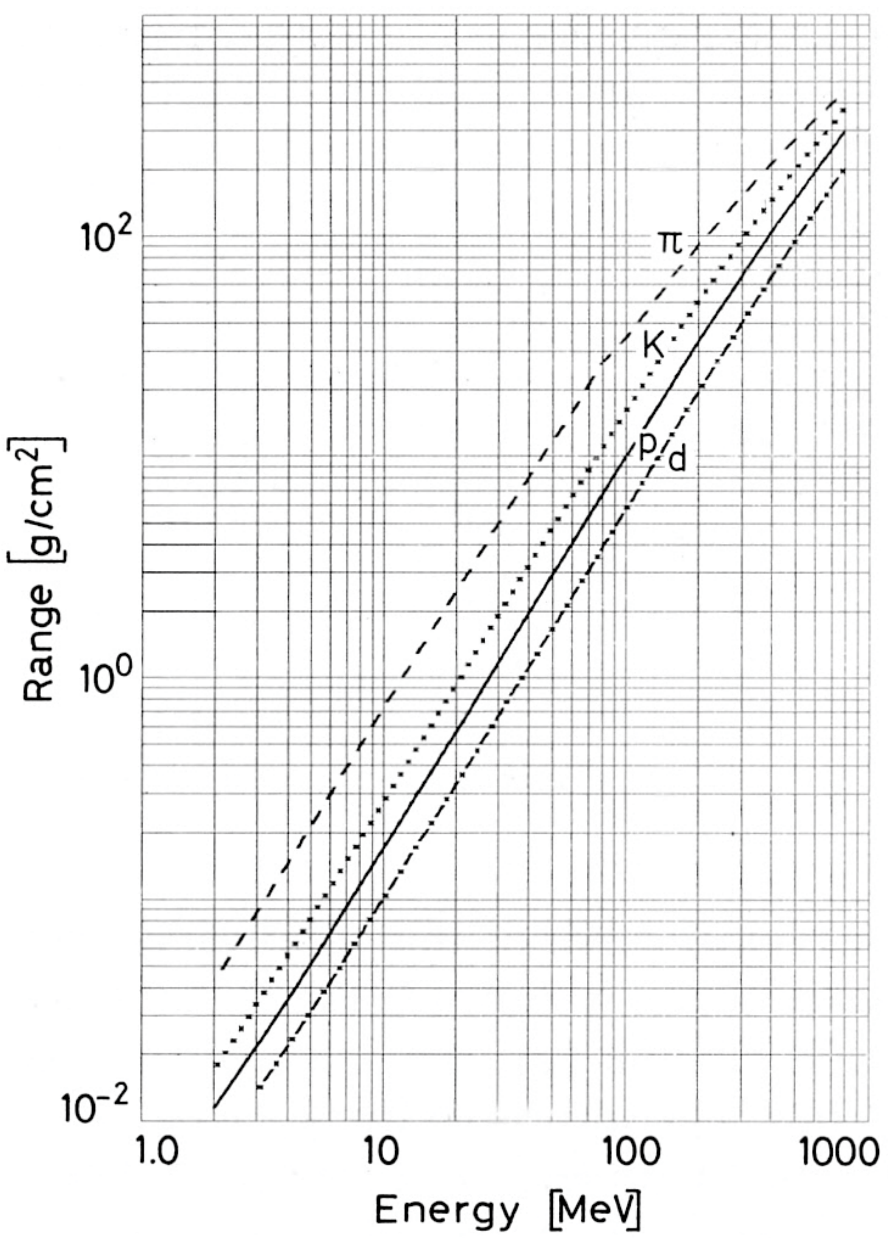
- Shape of curve due to energy loss fluctuations (folding with Gaussian)
- Extrapolated range (defined by extrapolation of tangent at T = 0.5)

- In general, range can be calculated by integration over energy loss curve (Bethe-Bloch formula, in the range of applicability)

Range R for a particle with kinetic energy T_0:

$$R(T_0) = \int_{T^{min}}^{T_0} \left(\frac{dE}{dx} \right)^{-1} \cdot dE + R_0(T^{min})$$

- Ignores the effect of multiple scattering (is however small)
- T^{min}: lower value of kinetic energy for which the Bethe-Bloch formula is valid



Range curves for different hadrons / nuclei as function of their kinetic energy

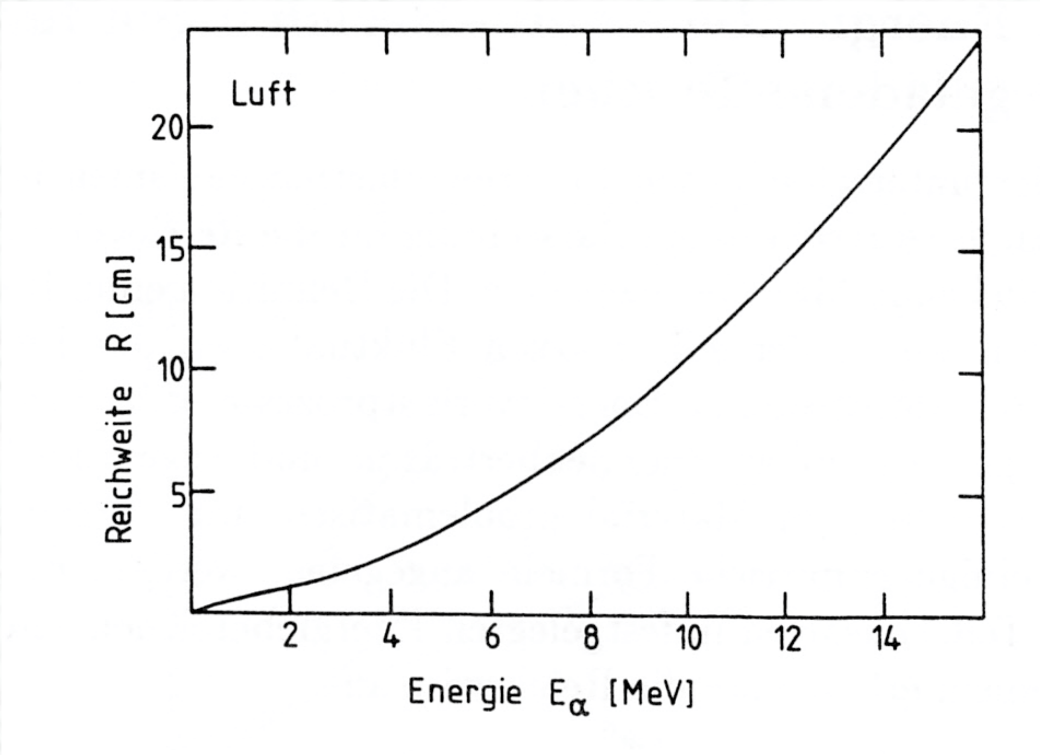
From measurements (nearly straigth lines in log-log plot) follows a power-law behaviour:

$R \sim E^{\,b}$

Bethe-Bloch: $dE/dx \sim 1/\beta^2$ → $b = 2$

Experiment: $1.5 < b < 2.0$

Range of α particles



Range of α particles in air

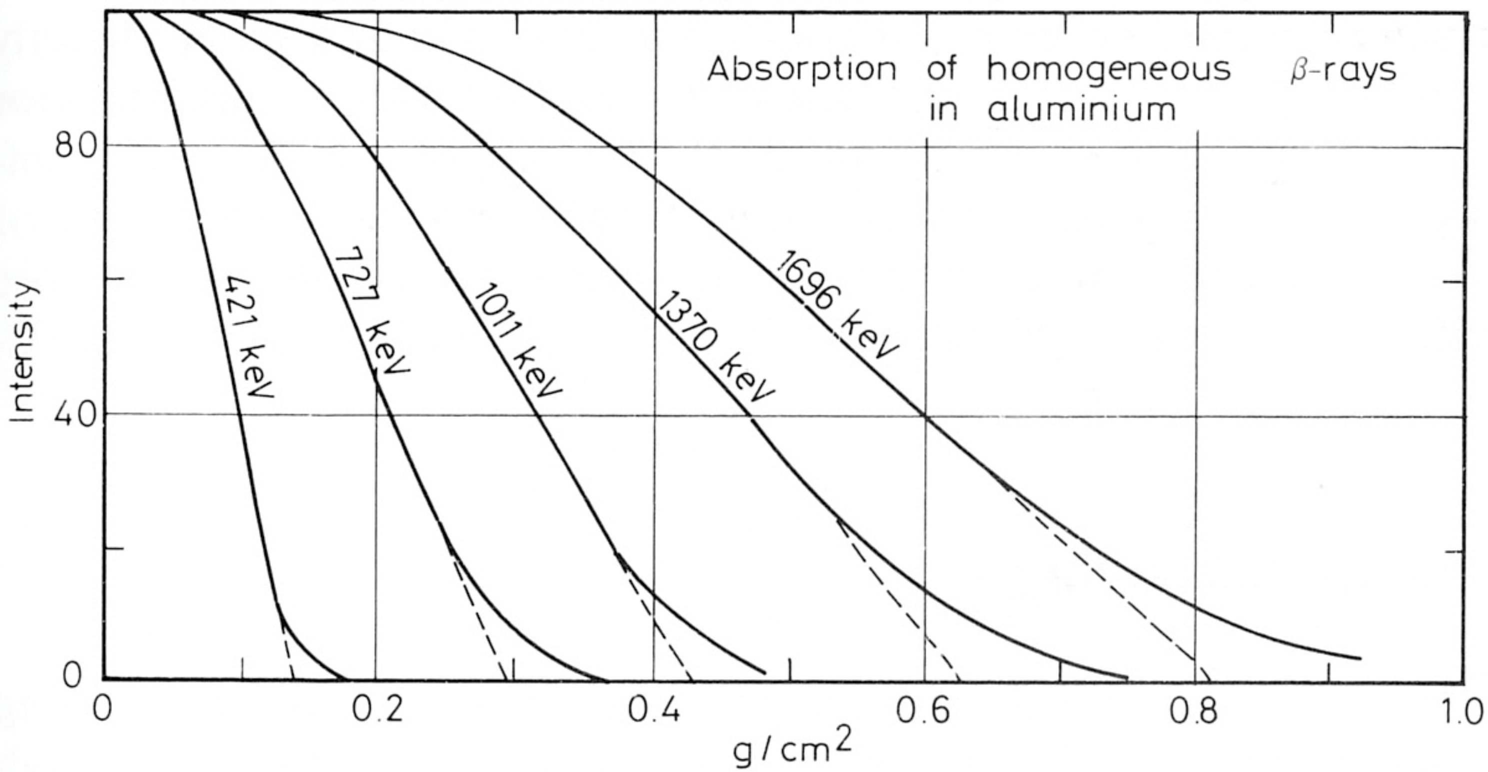
$R_\alpha = 0.31\, E_{kin}^{3/2}$

[E_{kin} in MeV, R_α in cm]

For other materials:

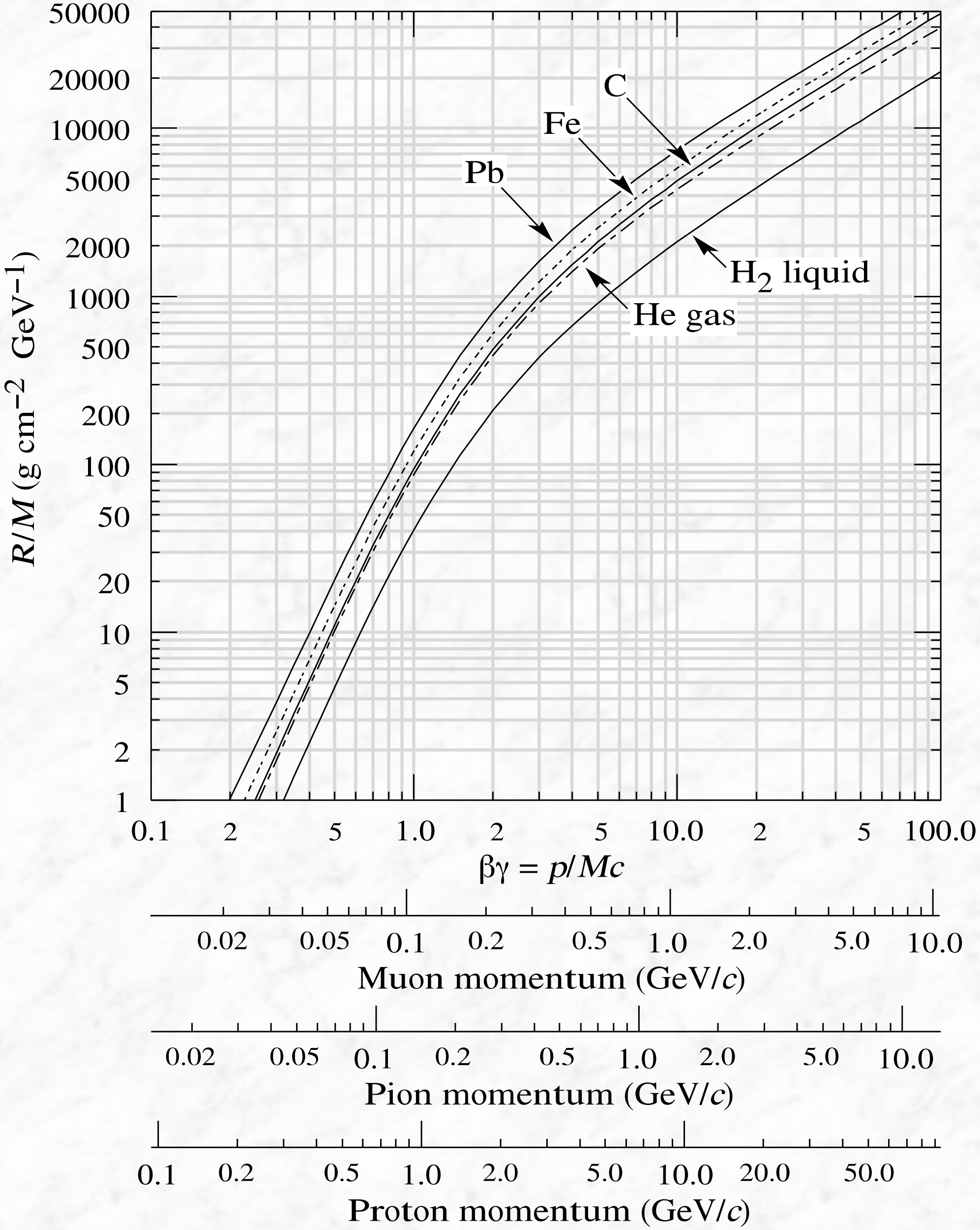
$R_\alpha = 3.2\, 10^{-4} \sqrt{A}/ \rho\, R_\alpha^{\,air}$

Range of electrons



◄ **Fig. 2.11.** Range number-distance curves for electrons (from *Marshall* and *Ward* [2.16])

Range of electrons (measured in aluminium)

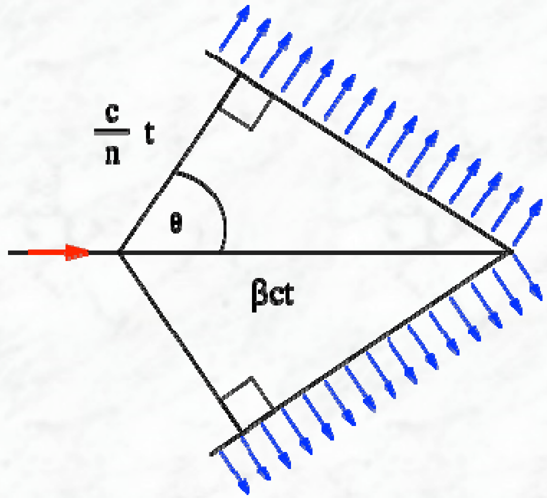


Range of heavy charged particles in liquid hydrogen, He gas, carbon, iron, and lead. (Particle Data Group, 2013).

2.4 Cherenkov Radiation

A charged particle that moves in a dielectric medium with a velocity v > c/n, i.e. has a velocity above the speed of light in this medium, emits a characteristic radiation, called Cherenkov radiation

1934 experimental discovery, P. Cherenkov
1937 theoretical explanation by Frank and Tamm



→ additional energy loss term:

$$-\frac{dE}{dx} = \left.\frac{dE}{dx}\right|_{ion} + \left.\frac{dE}{dx}\right|_{Brems} + \left.\frac{dE}{dx}\right|_{CH}$$

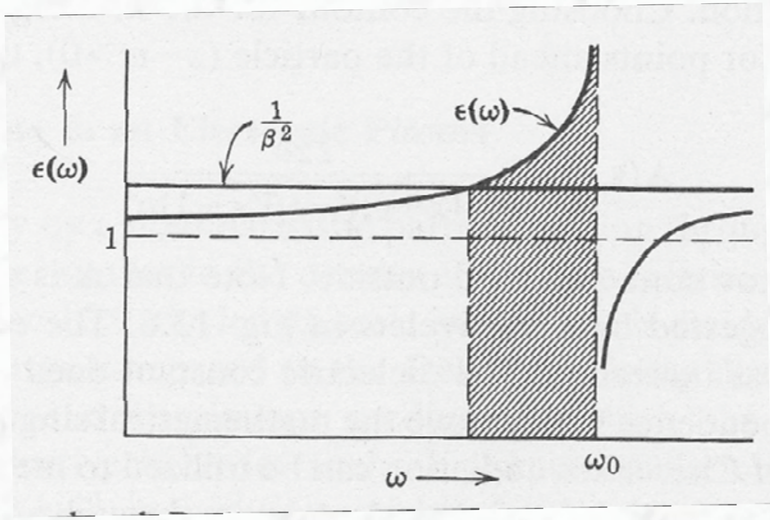
The energy loss contributions due to Cherenkov radiation are small, small correction of the order of percent to the ionization energy loss:

$$-\left.\frac{dE}{dx}\right|_{CH} \sim 0.01-0.02\ \mathrm{MeV/g\cdot cm^{-2}} \quad \text{(gases)}$$

$$\left.\frac{dE}{dx}\right|_{ion}^{min} \approx 1.5\ \mathrm{MeV/g\cdot cm^{-2}}$$

The requirement: $v > \frac{c}{n} \Leftrightarrow n > \frac{1}{\beta}$

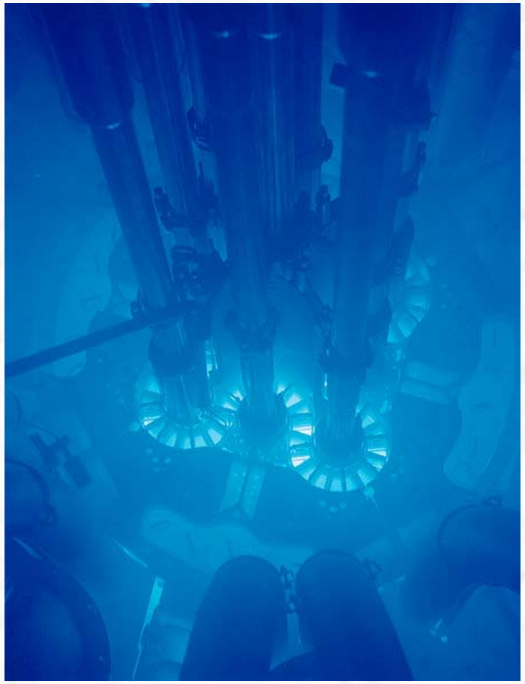
leads to a reduced frequency domain over which the emission of Cherenkov radiation is possible;
Explanation: dispersion relation *n(λ)* or *n(ω)*



Emission in optical to ultraviolet range
(→ blue colour of emitted radiation)

$$-\left.\frac{dE}{dx}\right|_{CH} = z^2 \cdot \frac{\alpha \cdot \hbar}{c} \cdot \int_{n(\omega)>1/\beta} \omega \cdot \left(1 - \frac{1}{\beta^2 n^2(\omega)}\right) \cdot d\omega$$

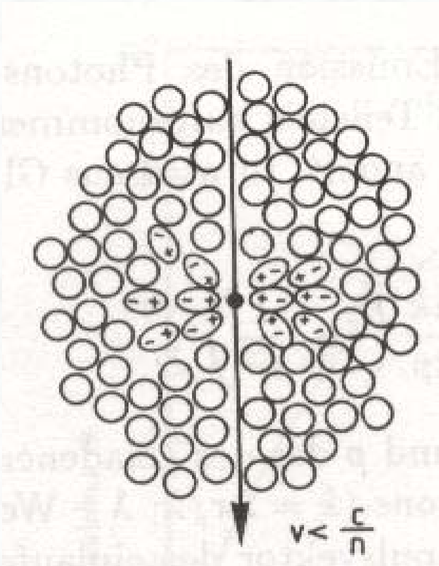
For calculation, see J.D. Jackson, Classical Electrodynamics



Cherenkov radiation in a nuclear reactor, Advanced Test Reactor, Idaho Nat. Labs

Ursprung der Strahlung: Polarisation des Mediums

$v < c/n$



Teilchen polarisiert das Medium

$v_{Pol} = c/n > v$

→ symmetrisch in Vorwärts- u. Rückwartsrichtung

→ kein resultierendes Dipolmoment

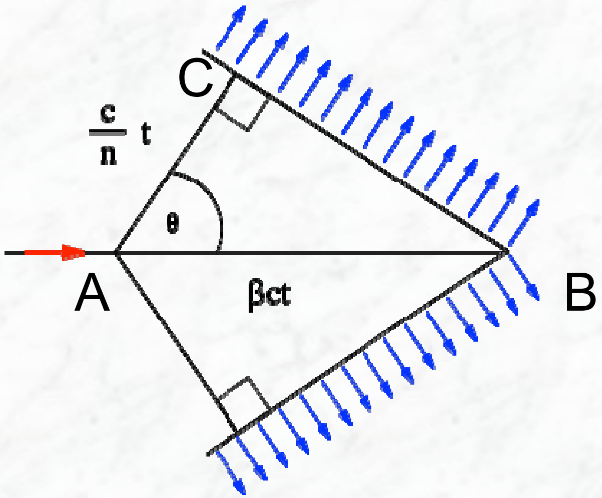
$v > c/n$

- Atome hinter dem Teilchen bleiben polarisiert
- keine Polarisation in Vorwärtsrichtung

→ resultierendes Dipolmoment am Ort des Teilchens

→ Strahlung

Geometrical consideration:



$s_1 = \overline{AB} = t \cdot \beta \cdot c$

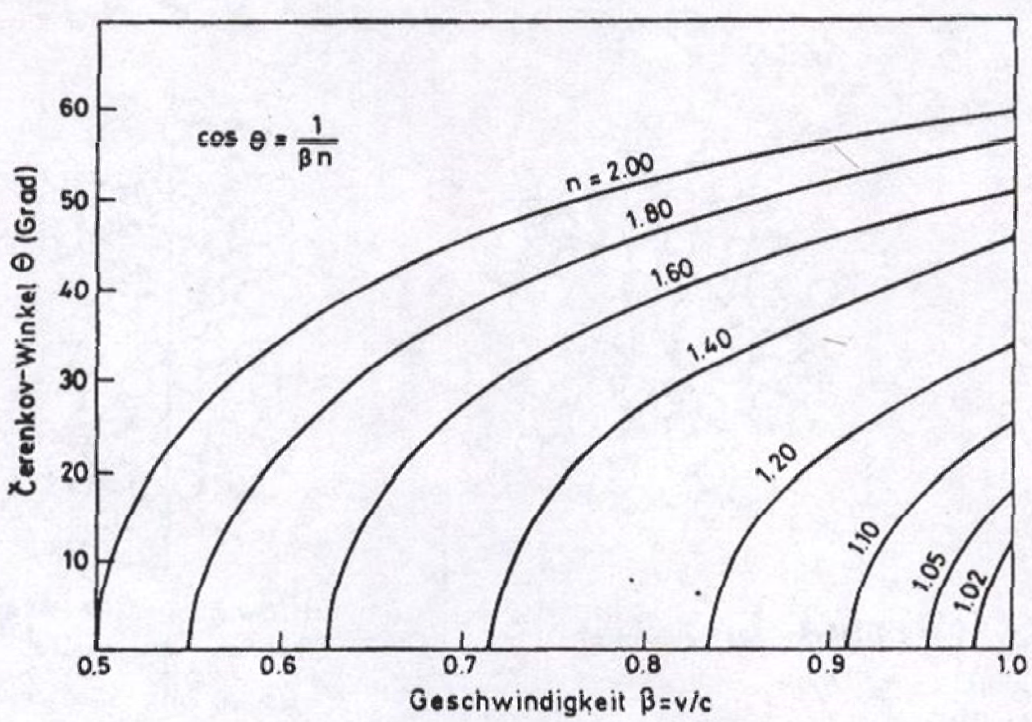
$s_2 = \overline{AC} = t \cdot c/n$

$$\cos\theta_c = \frac{s_2}{s_1} = \frac{1}{n \cdot \beta}$$

→ Threshold effect for the emission of Cherenkov radiation

$\cos\theta_c < 1 \quad \rightarrow \quad \beta > 1/n$

Dependence of the Cherenkov angle on β:



$$\theta_c^{min} = 0 \qquad \beta = 1/n$$

$$\frac{1}{n} < \beta < 1$$

$$\theta_c^{max} = \arccos\frac{1}{n} \qquad \beta = 1$$

Application: Particle Identification

(i) Threshold behaviour: light particles (electrons) emit radiation, heavier (mesons) not

(ii) Measurement of the Cherenkov angle → β (particle velocity)
In conjunction with additional measurements of p → particle mass

Refractive indices, Cherenkov threshold values

Material	$n-1$	β-Schwelle	γ-Schwelle
festes Natrium	3.22	0.24	1.029
Bleisulfit	2.91	0.26	1.034
Diamant	1.42	0.41	1.10
Zinksulfid ($ZnS(Ag)$)	1.37	0.42	1.10
Silberchlorid	1.07	0.48	1.14
Flintglas (SFS1)	0.92	0.52	1.17
Bleifluorid	0.80	0.55	1.20
Clerici-Lösung	0.69	0.59	1.24
Bleiglas	0.67	0.60	1.25
Thalliumformiat-Lösung	0.59	0.63	1.29
Szintillator	0.58	0.63	1.29
Plexiglas	0.48	0.66	1.33
Borsilikatglas	0.47	0.68	1.36
Wasser	0.33	0.75	1.52
Aerogel	0.025 - 0.075	0.93 - 0.976	4.5 - 2.7
Pentan (STP)	$1.7 \cdot 10^{-3}$	0.9983	17.2
CO_2 (STP)	$4.3 \cdot 10^{-4}$	0.9996	34.1
Luft (STP)	$2.93 \cdot 10^{-4}$	0.9997	41.2
H_2 (STP)	$1.4 \cdot 10^{-4}$	0.99986	59.8
He (STP)	$3.3 \cdot 10^{-5}$	0.99997	123

Tabelle 6.2: Cherenkov-Radiatoren [94, 32, 313]. Der Brechungsindex für Gase bezieht sich auf $0°C$ und $1\,atm$ (STP). Festes Natrium ist für Wellenlängen unterhalb von 2000 Å transparent [373, 209].

problematic: region between liquids and gases

Aerogel: mixture of
$m\,(SiO_2) + 2m\,(H_2O)$

light structure with inclusions of air, bubbles with diameter $< \lambda_{Licht}$

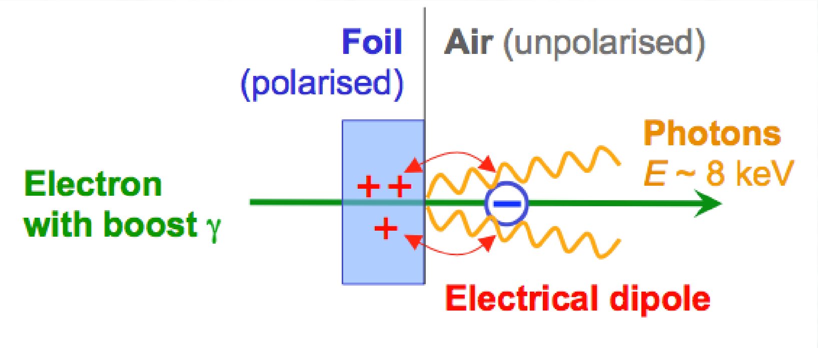
$\rightarrow$ n: average from n_{air}, n_{SiO2}, n_{H2O}

2.5 Transition Radiation

- When crossing boundaries of two media with different dielectrical constants, a charged particles emits electromagnetic radiation, transition radiation

  Reason: adaptation of the electric fields (ε_1, ε_2)

  1946 Discovery and explanation by Ginsburg and Frank
  (for a theoretical description, see Jackson, Classical Electrodynamics)



Formation of transition radiation occurs in a small region, at the boundary,
Formation length: D ≈ γ 10^{-6} cm

- The total emitted energy in form of transition radiation is proportional to the Lorentz factor γ

$$\left.\frac{dE}{dx}\right|_{ion}^{min} \approx 1.5\,\text{MeV}/\text{g}\cdot\text{cm}^{-2}$$

(c = 1)

  extremely important for the identification of particles in the high momentum / energy range (where $\beta \approx 1$)

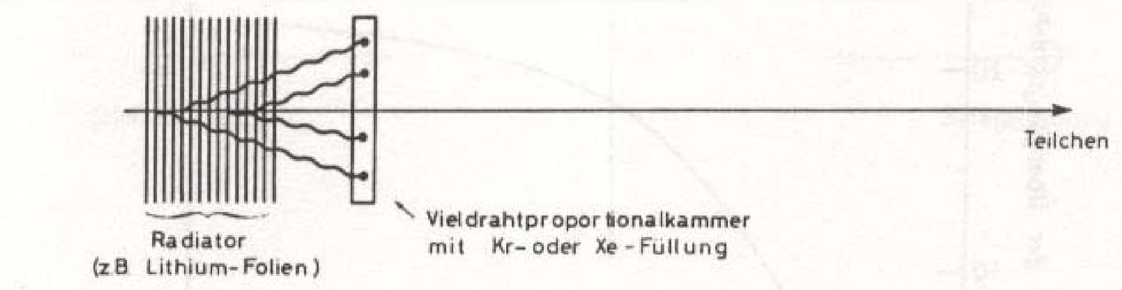
- The application of transition radiation detectors is mainly for the identification of electrons;

  For a given momentum p, their γ factor is much larger than for hadrons (factor 273 for the lightest charged hadron $\pi^{\pm}$)

$$\gamma = \frac{E}{m} = \frac{\sqrt{p^2 + m^2}}{m} \approx \frac{p}{m}$$

- For a γ value of 10^3 (e with p=0.5 GeV, $\pi^{\pm}$ with p ≈ 140 GeV) about half of the radiated energy is found in the Röntgen energy range (2 – 20 keV γ radiation)

  These γ quanta have to be detected, use absorber material with high Z value (absorption via photoelectric effect, see later, e.g. Xenon gas)



2.5 Strong interaction of hadrons

- Charged and neutral hadrons can interact with the detector material, in particular in the dense calorimeter material, via the strong interaction

- The relevant interaction processes are inelastic hadron-hadron collisions, e.g. inelastic πp, Kp, pp and np scattering; In such interactions, usually new hadrons (mesons) are created, energy is distributed to higher multiplicities

  Hadronic interactions are characterized by the hadronic interaction length λ_{had}

- A beam of hadrons is attenuated in matter due to hadronic interactions as

$$I(x) = I_0 e^{-x/\lambda_{had}}$$

where x = depth in material

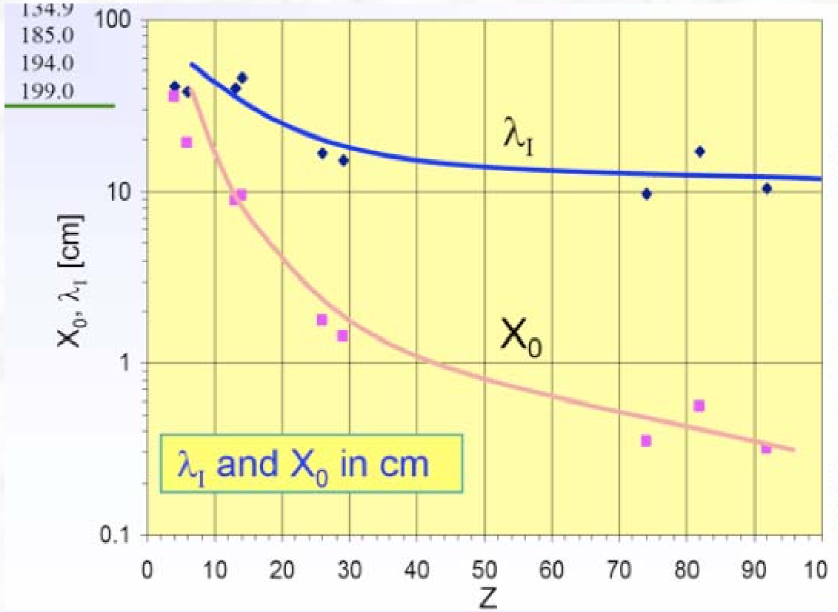
- The hadronic interaction length is a material constant and is linked to the inelastic interaction cross section σ_{inel}

$$\frac{1}{\lambda_{had}} = \sigma_{inel} \cdot \frac{N_A \cdot \rho}{A}$$

Approximation: $\lambda_{had} \approx 35\, A^{1/3}$ (cm)
note: in contrast to the radiation length X_0, the hadronic interaction length are large

Some values for radiation and hadronic absorption lengths:

Material	X_0 (cm)	λ_{had} (cm)
H_2 Gas	865	718
He	755	520
Be	35.3	40.7
C	18.8	38.1
Fe	1.76	16.76
Cu	1.43	15.06
W	0.35	9.59
Pb	0.56	17.09



note: for high Z materials, the hadronic interaction lengths are about a factor 10-30 larger than the radiation lengths

→ much more material is needed to stop hadrons compared to electrons; this explains the large extension of the hadronic calorimeters in large detector systems

Layers of the ATLAS detector