Problem set for the lecture

Particle Detectors, WS 2015/16

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Problem set 5

Deadline: Thursday November 26, 10am

(Please drop into mailbox number 1 on the ground floor of the Gustav-Mie building.)

1. **Electron drift in noble gases**
   Electrons drift in helium of T = 20° and p = 1013 mb, with the electrical field strength being small. For electron energies below 10 eV only elastic scattering of electrons with helium atoms takes place, with a cross-section $\sigma = 5 \cdot 10^{-20}$ m^2.
   (a) What is the mean free path of the electrons?
   (b) Assuming the thermic component of the electron velocity to dominate, what is their mean velocity and time between two interactions?

   (2 Points)

2. **Lorentz angle and drift**
   In a thin planar detector a voltage of 2 kV is applied between anode and cathode, which are 2 cm apart. In addition there is a magnetic field of 1 T perpendicular to the electrical field. All incident particles are assumed to arrive perpendicular to the detector, their path in good approximation being a straight line within the detector volume.
   (a) A Lorentz angle of 35° is observed between the direction of electron drift and the electrical field vector. Estimate the mean free path of electrons in the medium.
   (b) What is the change in maximum drift time of electrons with and without the magnetic field?
   (c) The area of charge deposition on the anode is used to measure the position of incident particles. At what angle should the detector be tilted to achieve the best spatial resolution (keeping the electric field perpendicular to the magnetic field)?
   (d) What is the Lorentz angle for positive ions, exploiting that their mobility can be expressed as $\mu = \frac{q}{m}\tau$, with τ being the mean time between two interactions and using the mobility $\mu = 1$ cm^{2}V^{-1}s^{-1}? If both the areas of charge collection of electrons and ions are to be used to determine the incident particle position, at what angle does the detector have to be tilted to have the centres of both areas overlap (seen from the direction of incident particles), allowing the use of a simple offset correction for the measured track?

   (4 Points)

3. **NUMERICAL SIMULATION - Multiple scattering**
An experiment wants to measure the position and direction of muons with a momentum of 10 GeV after they traverse a calorimeter, which for this exercise is assumed to effectively correspond to traversing a 2 m thick layer of lead. Neglecting ionization losses, the expected spread in observed position and direction compared to extrapolating their path before impacting the calorimeter is to be numerically (e.g. ROOT) estimated, for muons impacting perpendicular to the lead layer. Assume elastic Coulomb scattering with lead nuclei to be the dominant process, using the Rutherford cross-section:

$$\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2}{8\pi\epsilon_0 mv^2}\right)^2 \frac{1}{sin^4(\theta/2)}$$

Detailed instructions are:

(a) Implement the Rutherford cross-section as a ROOT function and plot it for angles larger than $0.1°$.

(b) "Quantize" the problem using a small-step approach, determining a distance traversed in lead for which the probability of a scattering at an angle larger than $0.1°$ is about 10%, neglecting the possibility of subsequent scattering inside that interval. Fill a histogram with probabilities for scattering angles of [$0.1°$-$0.2°$], [$0.2°$-$0.3°$], ... when traversing that distance in lead. All angles below $0.1°$ are to be taken as zero (no scattering).

(c) Using random numbers drawn from the histogram, simulate the path of muons through the lead layer, for the simplified case of planar geometry and keeping track of muon position and direction. Using a sufficiently high number of muons, what are the distributions of number of scatterings, distances of the exit points from the case of no scattering taking place, as well as the angles in respect to the original muon direction. What are the respective means?

(d) Is a simple Gaussian assumptions viable for the total "diffusion" and deflection? What is the situation for a lead layer one fifth or five times as thick?

(4 Points)