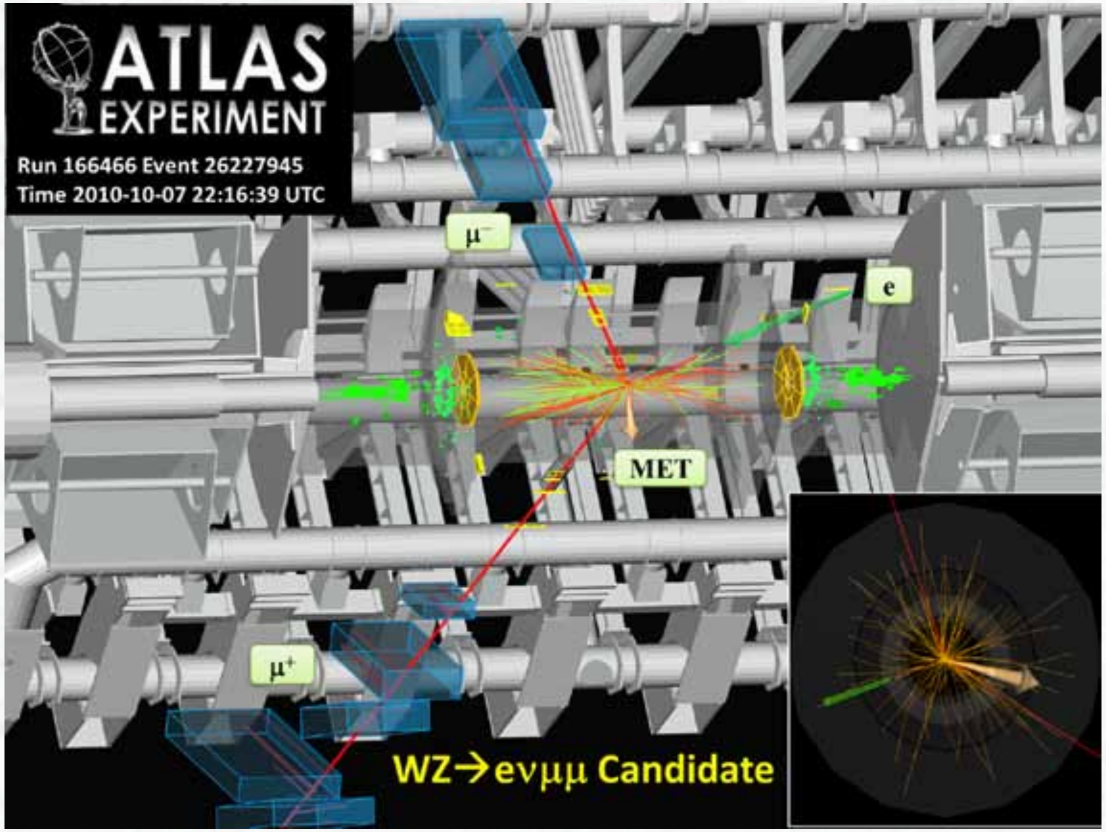
Part II: Detection Techniques and LHC detectors



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Part II: Detection techniques and LHC detectors

1. Interaction of charged particles with matter

2. Measurement of Charged Particles
   (Momentum measurements, tracking of charged particles,
   ATLAS and CMS detectors)

3. Energy measurement
   - Interaction of neutral particles with matter
   - Energy measurements in calorimeters

4. Muon detection at the LHC

5. Experimental conditions at the LHC (triggering, data acquisition,…)

1.1 Introduction to Detector Physics

- Detection through interaction of the particles with matter,
  e.g. via energy loss in a medium (ionization and excitation)

- Energy loss must be detected, made visible,
  mainly in form of electric or light signals

- Fundamental interaction for charged particles: electromagnetic interaction
  Energy is mainly lost due to interaction of the particles with the electrons of the atoms of the medium

  Cross sections are large: $\sigma \sim 10^{-17} - 10^{-16}$ cm^2 !!

  Small energy loss per collision, however, large number of them in dense materials

- Interaction processes (energy loss, scattering,..) are a nuisance for precise measurements and limit their accuracy

Overview on energy loss / detection processes

Charged particles	Photons, γ
Ionisation and excitation Bremsstrahlung	Photoelectric effect Compton scattering Pair creation
Cherenkov radiation Transition radiation	

1.2 Energy loss by ionisation and excitation

- A charged particle with mass m_0 interacts primarily with the electrons (mass m_e) of the atom;
  Inelastic collisions → energy loss

- Maximal transferable kinetic energy is given by: $T^{\max} = \dfrac{2m_e c^2\beta^2\gamma^2}{1+2\gamma m_e / m_0 + (m_e / m_0)^2}$

  max. values: Muon with E = 1.06 GeV (γ = 10): E_{kin}^{max} ~100 MeV

  Two types of collisions:
  Soft collision: only excitation of the atom
  Hard collision: ionisation of the atom
  In some of the hard collisions the atomic electron acquires such a large energy that it causes secondary ionisation (δ-electrons).

  → Ionisation of atoms along the track / path of the particle;
  In general, small energy loss per collision, but many collisions in dense materials → energy loss distribution
  one can work with average energy loss

- Elastic collisions from nuclei caus very little energy loss, they are the main cause for deflection / scattering under large angles

Bethe-Bloch Formula

Bethe-Bloch formula gives the mean rate of energy loss (stopping power) for a heavy charged particle ($m_0 >> m_e$)* (note:

$$-\frac{dE}{dx} = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

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A : atomic mass of absorber

$\frac{K}{A} = 4\pi N_A r_e^2 m_e c^2 / A = 0.307075 \text{ MeV g}^{-1}\text{cm}^2, \text{ for A} = 1\text{g mol}^{-1}$

z: atomic number of incident particle

Z: atomic number of absorber

I : energy needed for ionization

T^{max}: max. energy transfer (see previous slide)

δ $(\beta\gamma)$: density effect correction to inoization energy loss

$x = \rho$ s , surface density or mass thickness, with unit g/cm^2, where s is the length.

dE/dx has the units MeV cm^2/g

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* note: Bethe-Bloch formula is not valid for electrons (equal mass, identical particles)

History of Energy Loss Calculations: dE/dx

1915: Niels Bohr, classical formula, Nobel prize 1922.
1930: Non-relativistic formula found by Hans Bethe
1932: Relativistic formula by Hans Bethe

Bethe's calculation is leading order in pertubation theory, thus only z^2 terms are included.

Additional corrections:

- z^3 corrections calculated by Barkas-Andersen
- z^4 correction calculated by Felix Bloch
  (Nobel prize 1952, for nuclear magnetic resonance).
  Although the formula is called Bethe-Bloch formula the z^4 term is usually not included.
- Shell corrections: atomic electrons are not stationary
- Density corrections: by Enrico Fermi
  (Nobel prize 1938, for the discovery of nuclear reaction induced by slow neutrons).

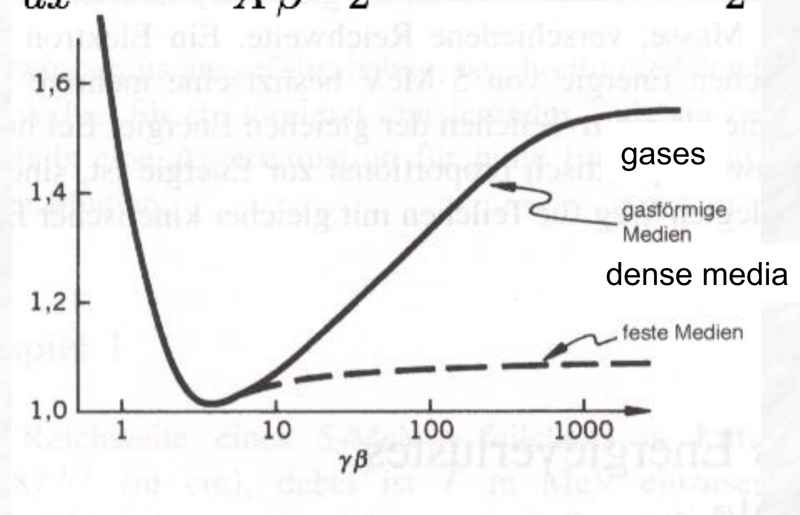
Hans Bethe (1906-2005)

Studied physics in Frankfurt and Munich, emigrated to US in 1933. Professor at Cornell U.,

Nobel prize 1967 for the theory of nuclear processes in stars.

Important features / dependencies:

$$-\frac{dE}{dx} = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} [\frac{1}{2} \ln f(\beta) - \beta^2 - \frac{\delta(\beta\gamma)}{2}]$$



- Energy loss is independent of the mass of the incoming particle
  → universal curve

- depends quadratically on the charge and velocity of the particle: $\sim z^2/\beta^2$

- dE/dx is relatively independent of the absorber (enters only via Z/A, which is constant over a large range of materials)

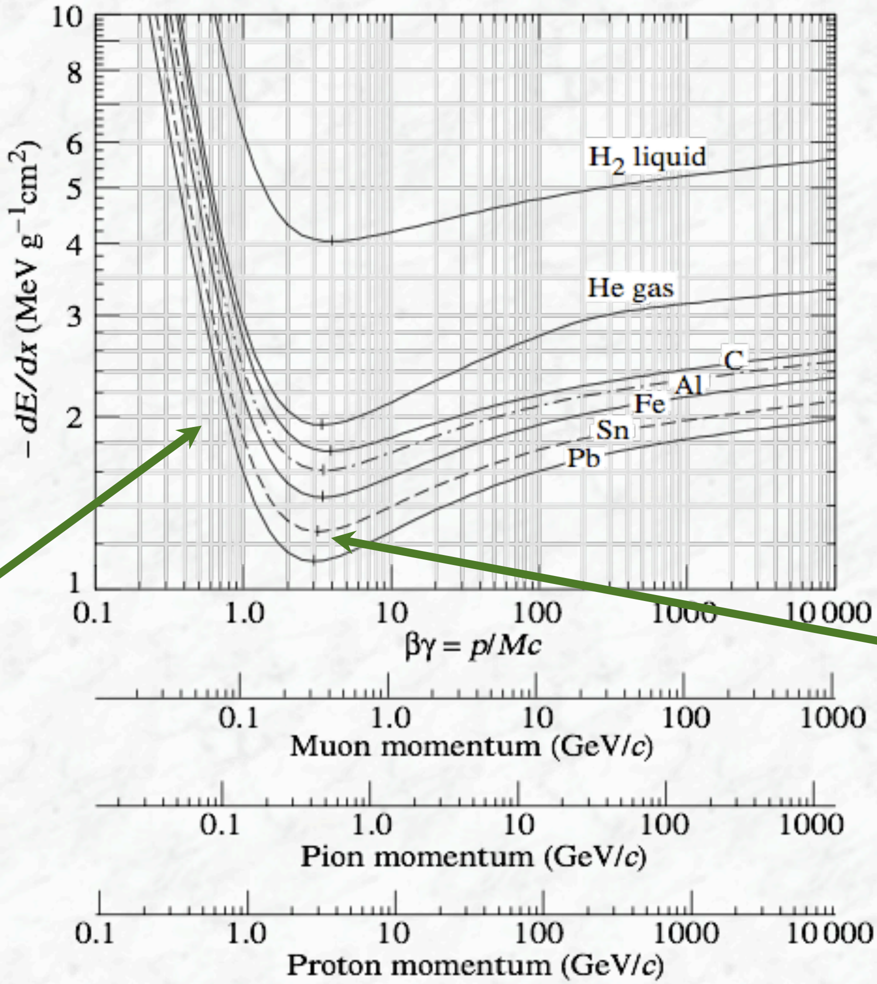
- Minimum for $\beta\gamma \approx 3.5$
  energy loss in the mimimum:

$$\boxed{\left.\frac{dE}{dx}\right|_{\min} \approx 1.5 \frac{MeV \cdot cm^2}{g}}$$

  (particles that undergo minimal energy loss are called "mimimum ionizing particle" = mip)

- Logarithmic rise for large values of $\beta\gamma$ due to relativistic effects is damped in dense media $\delta(\beta\gamma)$

Examples of Mean Energy Loss



$1/\beta^2$

Figure 27.3: Mean energy loss rate in liquid (bubble chamber) hydrogen, gaseous helium, carbon, aluminum, iron, tin, and lead. Radiative effects, relevant for muons and pions, are not included. These become significant for muons in iron for $\beta\gamma \gtrsim 1000$, and at lower momenta for muons in higher-Z absorbers. See Fig. 27.21.

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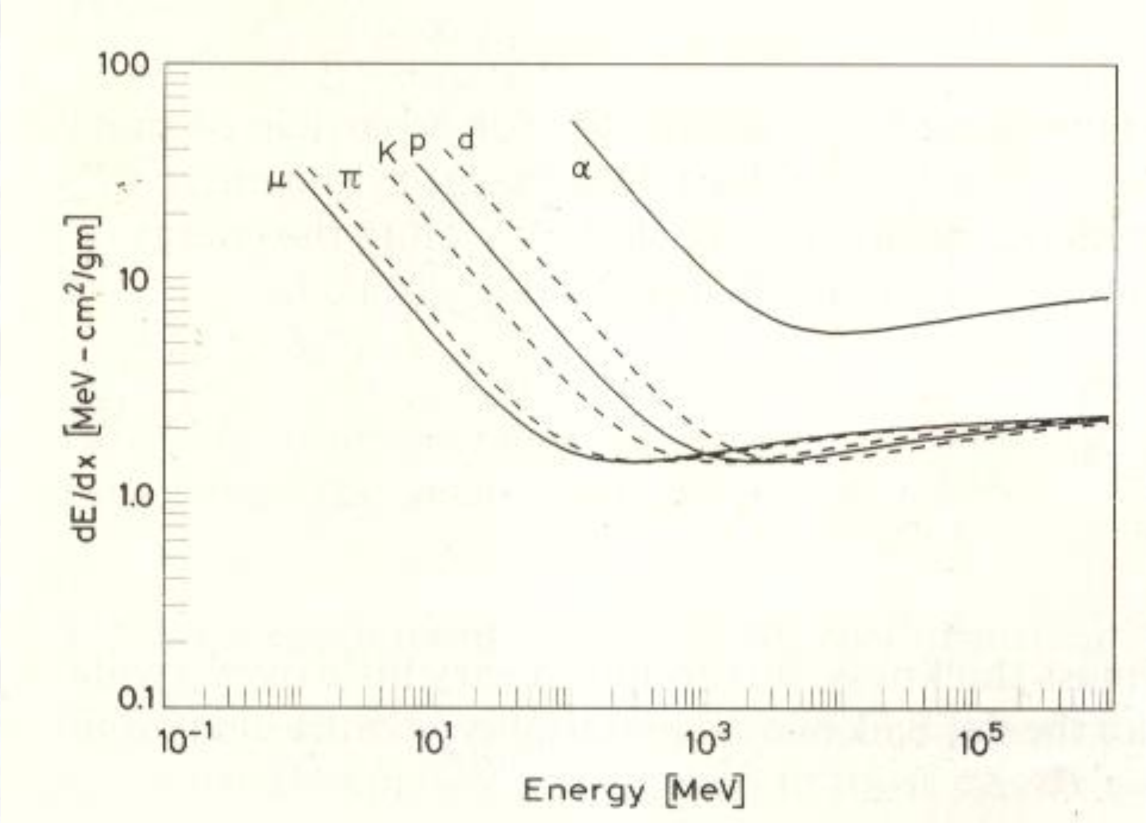
Bethe-Bloch formula:

$$-\frac{dE}{dx} = Kz^2\frac{Z}{A}\frac{1}{\beta^2}\left[\frac{1}{2}\ln f(\beta) - \beta^2 - \frac{\delta(\beta\gamma)}{2}\right]$$

Except in hydrogen, particles of the same velocity have similar energy loss in different materials.

The minimum in ionisation occurs at βγ = 3.5 to 3.0, as Z goes from 7 to 100

Consequence: dE/dx measurements can be used to identify particles

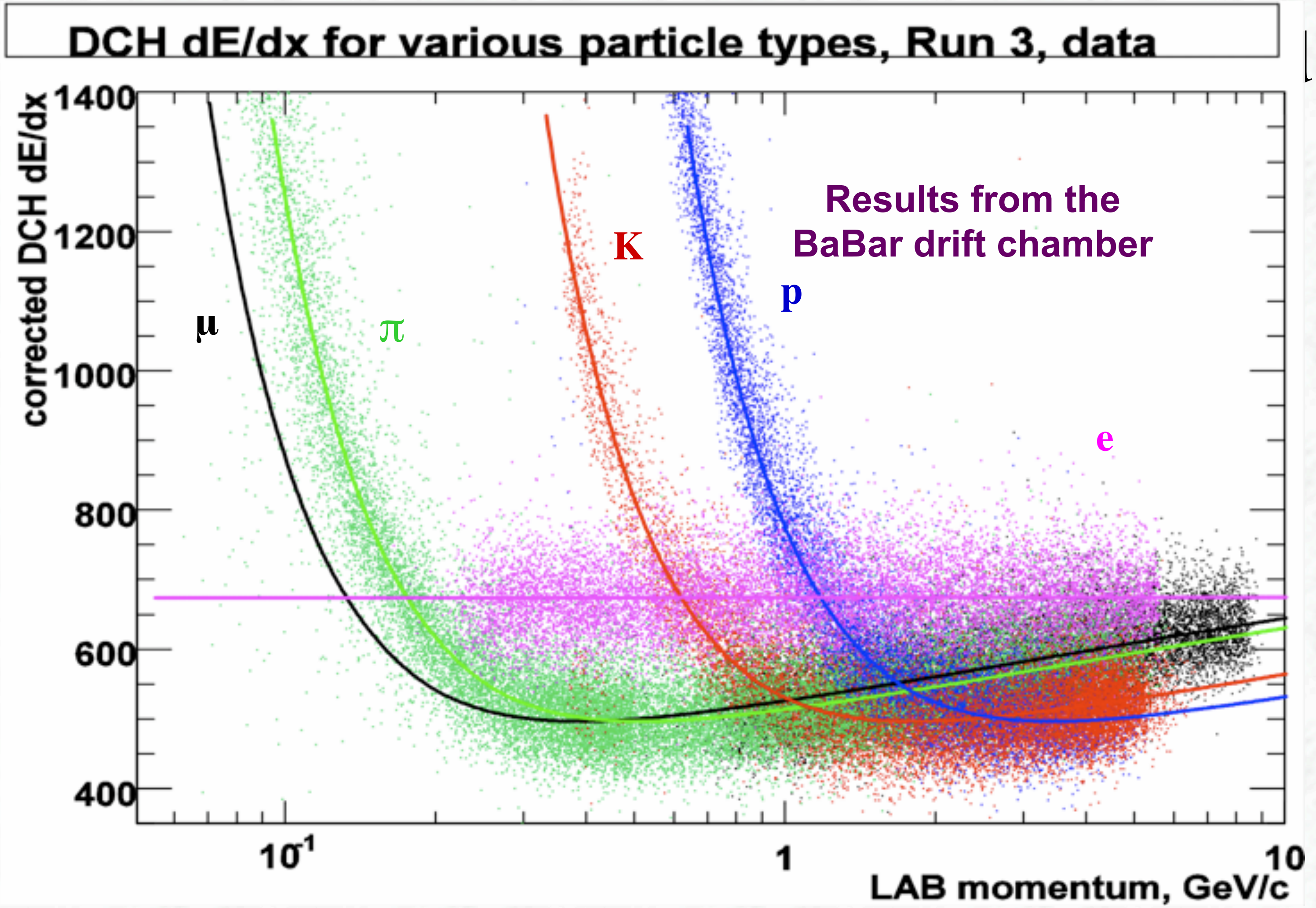


$$\beta\gamma = \frac{p}{E}\frac{E}{m} = \frac{p}{m}$$

- universal curve as function of $\beta\gamma$ splits up for different particle masses, if taken as function of energy or momentum

→ a simultaneous measurement of dE/dx and p,E → particle ID

Example: BaBar experiment at SLAC



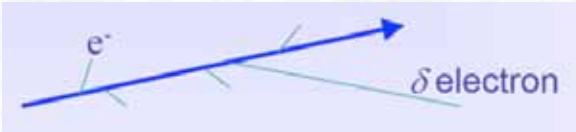
A simultaneous measurement of dE/dx and momentum p can provide particle identification. Works well in the low momentum range (< ~1 GeV)

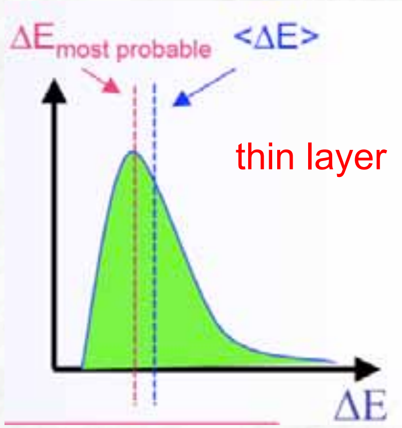
Fluctuations in Energy Loss

- A real detector (limited granularity) cannot measure <dE/dx>;
- It measures the energy ΔE deposited in layers of finite thickness Δx;

- Repeated measurements → sampling from an energy loss distribution

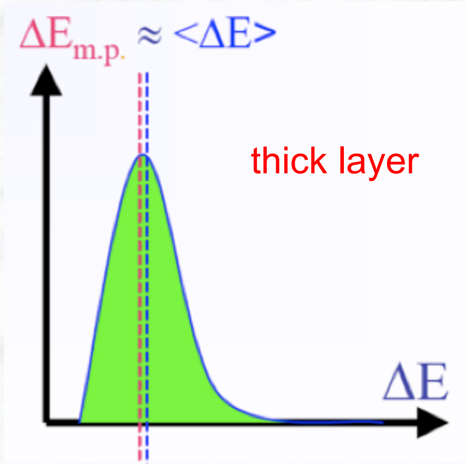
- For thin layers or low density materials, the energy loss distribution shows large fluctuations towards high losses, so called Landau tails.





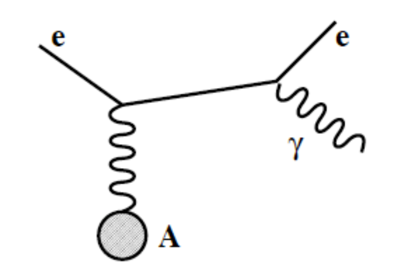
Example: Silicon sensor, 300 μm thick, ΔE_{mip} ~82 keV, $\langle \Delta E \rangle$ ~115 keV

- For thick layers and high density materials, the energy loss distribution shows a more Gaussian-like distribution (many collisions, Central llimit theorem)



1.3 Energy loss due to bremsstrahlung

- High energy charged particles undergo an additional energy loss (in addition to ionization energy loss) due to bremsstrahlung, i.e. radiation of photons, in the Coulomb field of the atomic nuclei



$$-\left.\frac{dE}{dx}\right|_{Brems} = 4\alpha N_A \left(\frac{e^2}{mc^2}\right)^2 \ln\frac{183}{Z^{1/3}} \frac{Z(Z+1)}{A} Q^2 E$$

where: Q, m = electric charge and mass of the particle,
α = fine structure constant
A,Z = atomic number, number of protons of the material
N_A= Avogardo’s number

- One can introduce the so-called radiation length X_0 defined via the above equation:

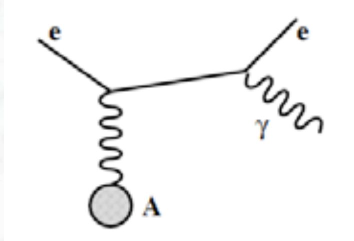
$$\frac{1}{X_0} = 4\alpha N_A \left(\frac{e^2}{m_e c^2}\right)^2 \ln\frac{183}{Z^{1/3}} \frac{Z(Z+1)}{A}$$

$$\xrightarrow{\text{for electrons}}$$

$$-\left.\frac{dE}{dx}\right|_{Brems} := \frac{1}{X_0} E$$

note that in the definition of X_0 the electron mass is used (electron as incoming particle). It only depends on electron and material constants and chacterises the radiation of electrons in matter

$$-\left.\frac{dE}{dx}\right|_{Brems} = 4\alpha N_A \left(\frac{e^2}{mc^2}\right)^2 \ln\frac{183}{Z^{1/3}} \frac{Z(Z+1)}{A} Q^2 E$$



Most important dependencies:

- material dependence $$\frac{dE}{dx} \sim \frac{Z(Z+1)}{A}$$

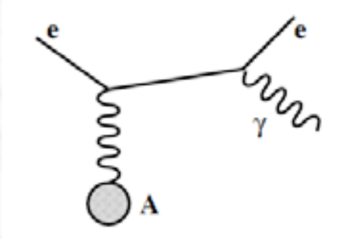
- depends on the mass of the incoming particle: $$\frac{dE}{dx} \sim \frac{1}{m^2}$$
  (light particles radiate more)
  This is the reason for the strong difference in bremsstrahlung energy loss between electrons and muons

$$\boxed{\left(\frac{dE}{dx}\right)_\mu \Big/ \left(\frac{dE}{dx}\right)_e \sim \frac{1}{40.000}}$$

- proportional to the energy of the incoming particle $$\frac{dE}{dx} \sim E$$

This implies that this energy loss contribution will become significant for high energy muons as well

For electrons the energy loss equation reduces to



$$-\left.\frac{dE}{dx}\right|_{Brems} := \frac{1}{X_0} E \quad \Rightarrow \quad E(x) = E_0 e^{-x/X_0}$$

- The energy of the particle decreases exponentially as a function of the thickness x of the traversed material, due to bremsstrahlung;

After $x=X_0$:
$$E(X_0) = \frac{E_0}{e} = 0.37 E_0$$

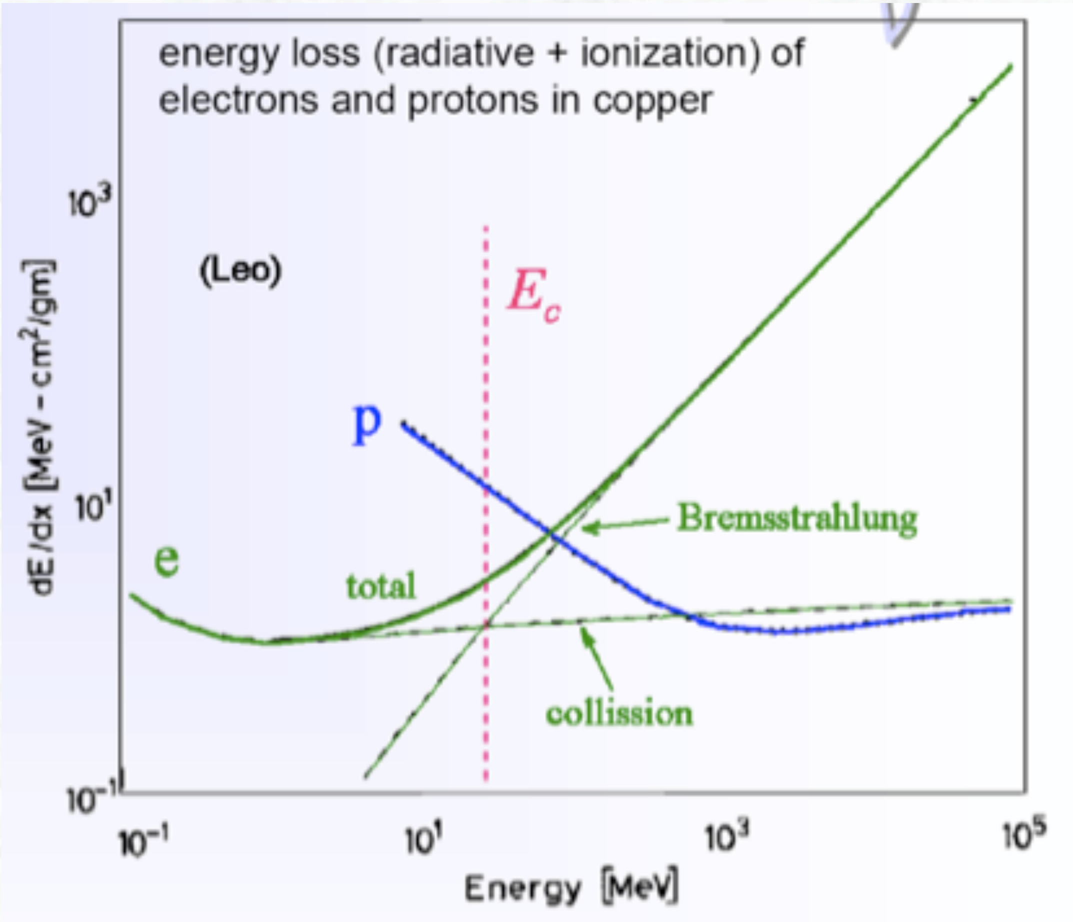
- Continuous 1/E energy loss spectrum, mainly soft radiation, with hard tail
- One defines the critical energy, as the energy where the energy loss due to ionization and bremsstrahlung are equal

$$-\left.\frac{dE}{dx}\right|_{ion} (E_c) = -\left.\frac{dE}{dx}\right|_{brems} (E_c)$$

useful approximations for electrons:
(heavy elements)

$$E_c = \frac{550\,\mathrm{MeV}}{Z}$$

$$X_0 = 180 \frac{A}{Z^2} \left(\frac{\mathrm{g}}{\mathrm{cm}^2} \right)$$

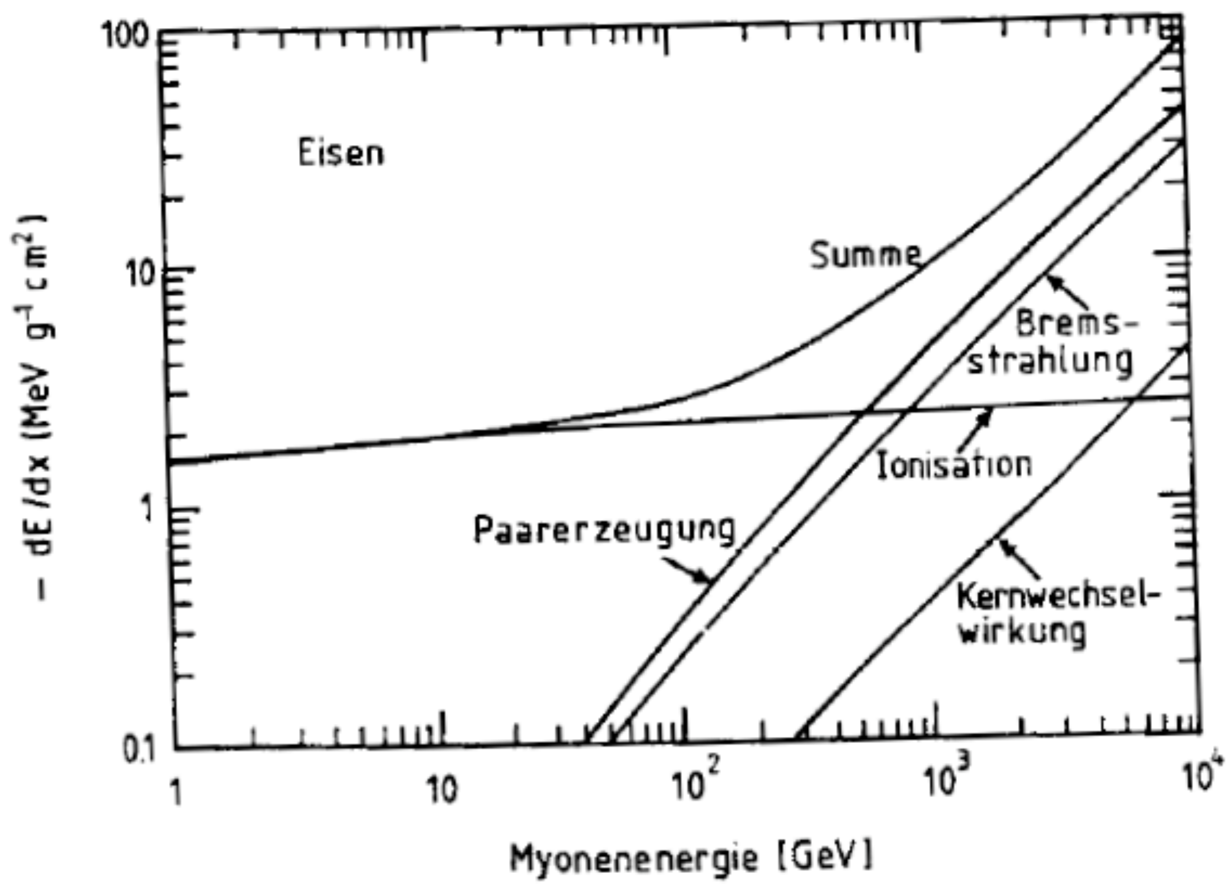


Critical energies in copper (Z = 29):

E_c (e) ≈ 20 MeV

$E_c(\mu)$ ≈ 1 TeV

- Muons with energies > ~10 GeV are able to penetrate thick layers of matter, e.g. calorimeters;
- This is the key signature for muon identification



Energy loss dE/dx for muons in iron;

- critical energy ≈ 870 GeV;

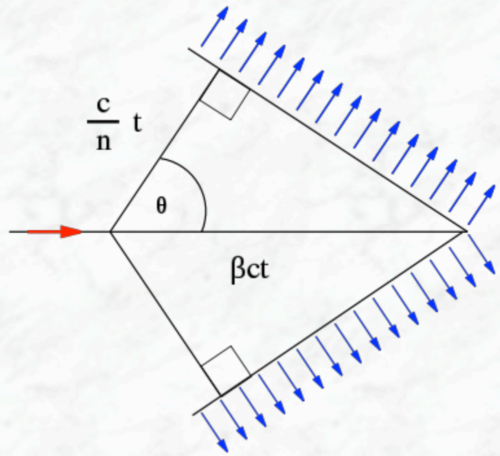
- At high energies also the pair creation μ (A) → μ e^+e^- (A) becomes important

Material	Z	X_0 (cm)	E_c (MeV)
H_2 Gas	1	700000	350
He	2	530000	250
Li	3	156	180
C	6	18.8	90
Fe	26	1.76	20.7
Cu	29	1.43	18.8
W	74	0.35	8.0
Pb	82	0.56	7.4
Air	7.3	30000	84
SiO_2	11.2	12	57
Water	7.5	36	83

Radiations lengths and critical energies for various materials
(from Ref. [Grupen])

1.4 Cherenkov Radiation

A charged particle that moves in a dielectric medium with a velocity v > c/n, i.e. has a velocity above the speed of light in this medium, emits a characteristic radiation, called Cherenkov radiation



1934 experimental discovery, P. Cherenkov
1937 theoretical explanation by Frank u. Tamm

→ additional energy loss term:

$$-\frac{dE}{dx} = \left.\frac{dE}{dx}\right|_{ion} + \left.\frac{dE}{dx}\right|_{Brems} + \left.\frac{dE}{dx}\right|_{CH}$$

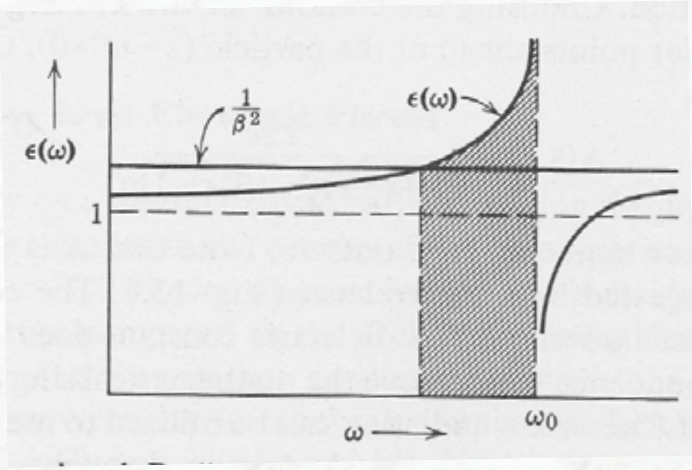
The energy loss contributions due to Cherenkov radiation are small, small correction of the order of percent to the ionization energy loss:

$$-\left.\frac{dE}{dx}\right|_{CH} \sim 0.01-0.02\ \text{MeV}/\text{g}\cdot\text{cm}^{-2} \quad \text{(gases)}$$

$$\left.\frac{dE}{dx}\right|_{ion}^{min} \approx 1.5\,\text{MeV}/\text{g}\cdot\text{cm}^{-2}$$

The requirement: $v > \frac{c}{n} \quad \Leftrightarrow \quad n > \frac{1}{\beta}$

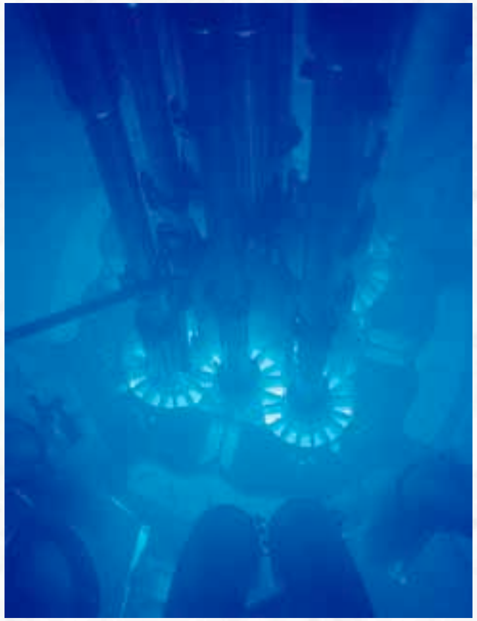
leads to a reduced frequency domain over which the emission of Cherenkov radiation is possible;
Explanation: dispersion relation *n(λ)* or *n(ω)*



Emission in optical to ultraviolet range
(blue colour of emitted radiation)

$$-\frac{dE}{dx}\bigg|_{CH} = Z^2 \cdot \frac{\alpha \cdot h}{c} \cdot \int_{n(\omega)>1/\beta} \omega \cdot \left(1 - \frac{1}{\beta^2 n^2(\omega)}\right) \cdot d\omega$$

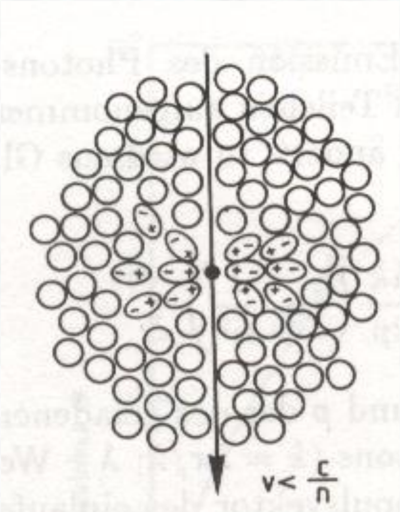
For calculation, see J.D. Jackson, Classical Electrodynamics



Cherenkov radiation in a nuclear reactor, Advanced Test Reactor, Idaho Nat. Labs

Ursprung der Strahlung: Polarisation des Mediums

$v < {}^{c}\!/_{n}$



$v > {}^{c}\!/_{n}$

Teilchen polarisiert das Medium

$v_{Pol} = c/n > v$

→ symmetrisch in Vorwärts- u. Rückwartsrichtung

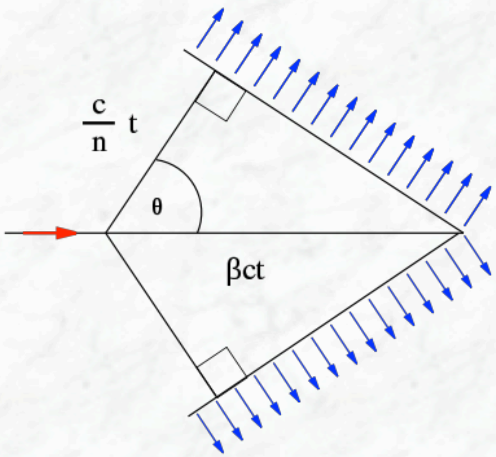
→ kein resultierendes Dipolmoment

- Atome hinter dem Teilchen bleiben polarisiert
- keine Polarisation in Vorwärtsrichtung

→ resultierendes Dipolmoment am Ort des Teilchens

→ Strahlung

Geometrical consideration:



$s_1 = \overline{AB} = t \cdot \beta \cdot c$

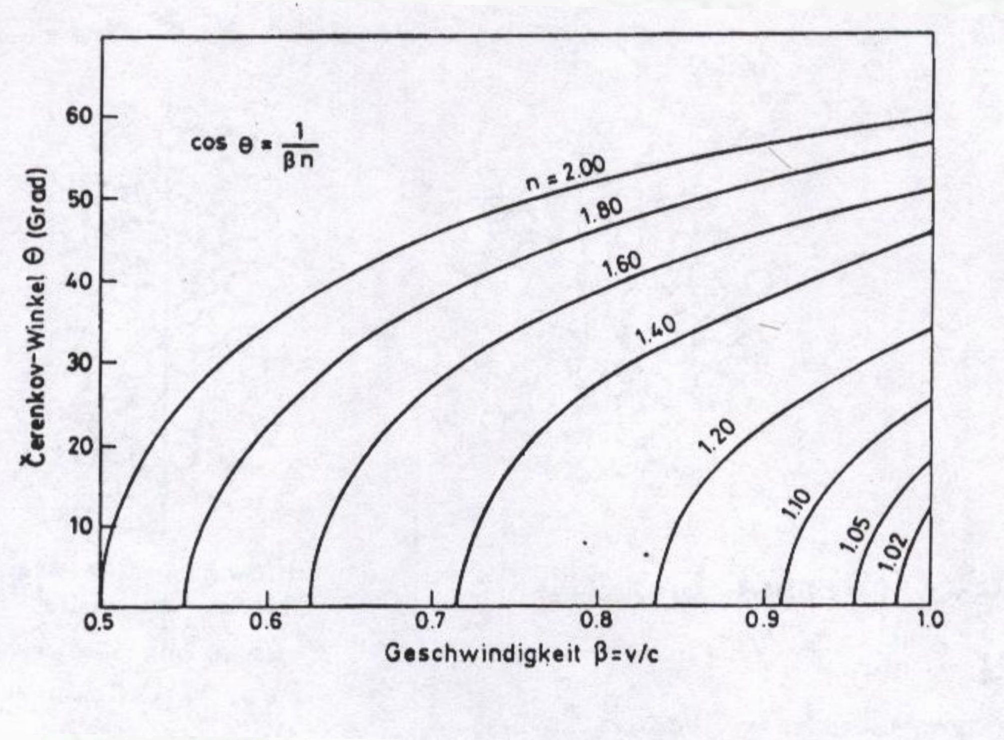
$s_2 = \overline{AC} = t \cdot c/n$

$$\boxed{\cos\theta_c = \frac{s_2}{s_1} = \frac{1}{n \cdot \beta}}$$

→ Threshold effect for the emission of Cherenkov radiation

$\cos\theta_c < 1 \quad \rightarrow \quad \beta > 1/n$

Dependence of the Cherenkov angle on β:



$$\theta_c^{min} = 0 \qquad \beta = 1/n$$

$$\frac{1}{n} < \beta < 1$$

$$\theta_c^{max} = \arccos\frac{1}{n} \qquad \beta = 1$$

Application: Particle Identification

(i) Threshold behaviour: light particle (electrons) emit radiation, heavier (mesons) not

(ii) Measurement of the Cherenkov angle → β (particle velocity)
In conjunction with additional measurements of p → particle mass

Refractive indices, Cherenkov threshold values

Material	$n-1$	β-Schwelle	γ-Schwelle
festes Natrium	3.22	0.24	1.029
Bleisulfit	2.91	0.26	1.034
Diamant	1.42	0.41	1.10
Zinksulfid ($ZnS(Ag)$)	1.37	0.42	1.10
Silberchlorid	1.07	0.48	1.14
Flintglas (SFS1)	0.92	0.52	1.17
Bleifluorid	0.80	0.55	1.20
Clerici-Lösung	0.69	0.59	1.24
Bleiglas	0.67	0.60	1.25
Thalliumformiat-Lösung	0.59	0.63	1.29
Szintillator	0.58	0.63	1.29
Plexiglas	0.48	0.66	1.33
Borsilikatglas	0.47	0.68	1.36
Wasser	0.33	0.75	1.52
Aerogel	0.025 - 0.075	0.93 - 0.976	4.5 - 2.7
Pentan (STP)	$1.7 \cdot 10^{-3}$	0.9983	17.2
CO_2 (STP)	$4.3 \cdot 10^{-4}$	0.9996	34.1
Luft (STP)	$2.93 \cdot 10^{-4}$	0.9997	41.2
H_2 (STP)	$1.4 \cdot 10^{-4}$	0.99986	59.8
He (STP)	$3.3 \cdot 10^{-5}$	0.99997	123

Tabelle 6.2: Cherenkov-Radiatoren [94, 32, 313]. Der Brechungsindex für Gase bezieht sich auf $0°C$ und $1\,atm$ (STP). Festes Natrium ist für Wellenlängen unterhalb von 2000 Å transparent [373, 209].

problematic: region between liquids and gases

Aerogel: mixture of
$m\,(SiO_2) + 2m\,(H_2O)$

light structure with inclusions of air, bubbles with diameter $< \lambda_{Licht}$

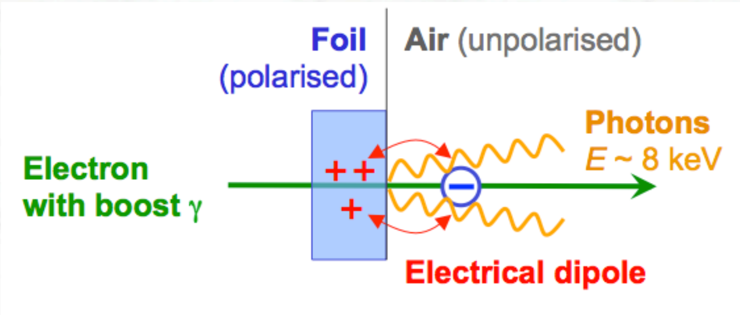
$\rightarrow$ n: average from n_{air}, n_{SiO2}, n_{H2O}

1.5 Transition Radiation

- When crossing boundaries of two media with different dielectron constants , a charged particles emits electromagnetic radiation, transition radiation

  Reason: adaptation of the electric fields (ε_1, ε_2)

  1946 Discovery and explanation by Ginsburg and Frank
  (for a theoretical description, see Jackson, Classical Electrodynamics)



Formation of transition radiation occurs in a small region, at the boundary,
Formation length: D ≈ γ 10^{-6} cm

- The total emitted energy in form of transition radiation is proportional to the Lorentz factor γ

$$\gamma = \frac{E}{m} = \frac{\sqrt{p^2 + m^2}}{m} \approx \frac{p}{m}$$

(c = 1)

  extremely important for the identification of particles in the high momentum / energy range (where $\beta \approx 1$)

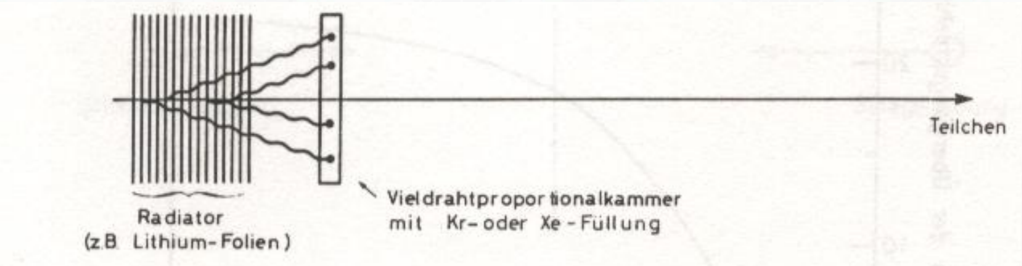
- The application of transition radiation detectors is mainly for the identification of electrons;

  For a given momentum p, their γ factor is much larger than for hadrons (factor 273 for the lightest charged hadron $\pi^{\pm}$)

$$\gamma = \frac{E}{m} = \frac{\sqrt{p^2 + m^2}}{m} \approx \frac{p}{m}$$

- For a γ value of 10^3 (e with p=0.5 GeV, $\pi^{\pm}$ with p ≈ 140 GeV) about half of the radiated energy is found in the Röntgen energy range (2 – 20 keV γ radiation)

  These γ quanta have to be detected, use absorber material with high Z value (absorption via photoelectric effect, see later, e.g. Xenon gas)



1.6 Strong interaction of hadrons

- Charged and neutral hadrons can interact with the detector material, in particular in the dense calorimeter material, via the strong interaction

- The relevant interaction processes are inelastic hadron-hadron collisions, e.g. inelastic πp, Kp, pp and np scattering;
  In such interactions, usually new hadrons (mesons) are created, energy is distributed to higher multiplicities

  Hadronic interactions are characterized by the hadronic interaction length λ_{had}

- A beam of hadrons is attenuated in matter due to hadronic interactions as

$$I(x) = I_0 e^{-x/\lambda_{had}}$$

where x = depth in material

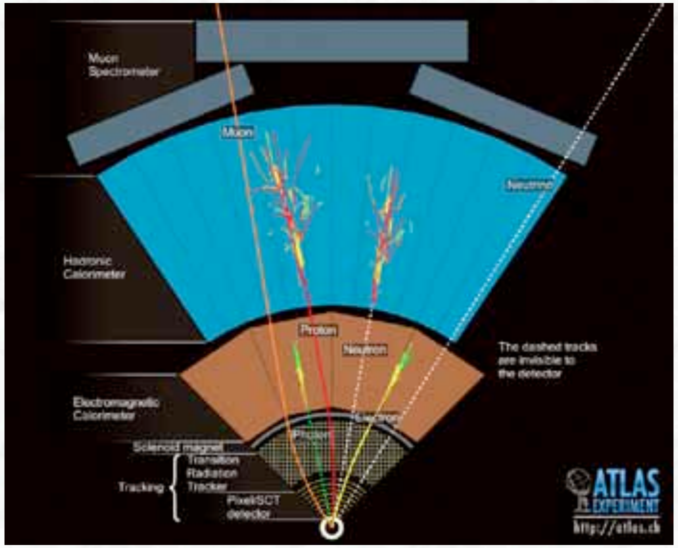
- The hadronic interaction length is a meterial constant and is linked to the inelastic interaction cross section σ_{inel}

$$\frac{1}{\lambda_{had}} = \sigma_{inel} \cdot \frac{N_A \cdot \rho}{A}$$

Approximation: $\lambda_{had} \approx 35\, A^{1/3}$ (cm)
note: in constrast to the radiation lenght X_0,
the hadronic interaction lenght are large (m range)

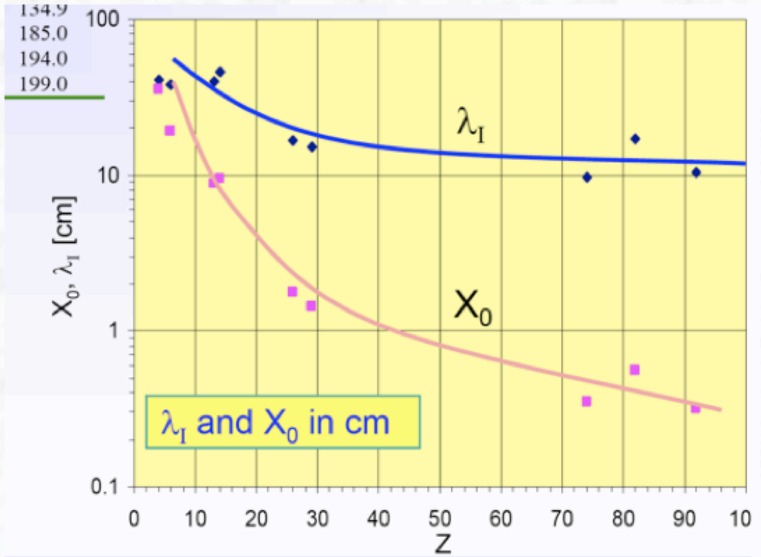
Some values for radiation and hadronic absorption lengths:

Material	X_0 (cm)	λ_{had} (cm)
H_2 Gas	865	718
He	755	520
Be	35.3	40.7
C	18.8	38.1
Fe	1.76	16.76
Cu	1.43	15.06
W	0.35	9.59
Pb	0.56	17.09



note: for high Z materials, the hadronic interaction lengths are about a factor 10-30 larger than the radiation lengths

→ much more material is needed to stop hadrons compared to electrons; this explains the large extension of the hadronic calorimeters



Part II, 2. Measurement of charged particles

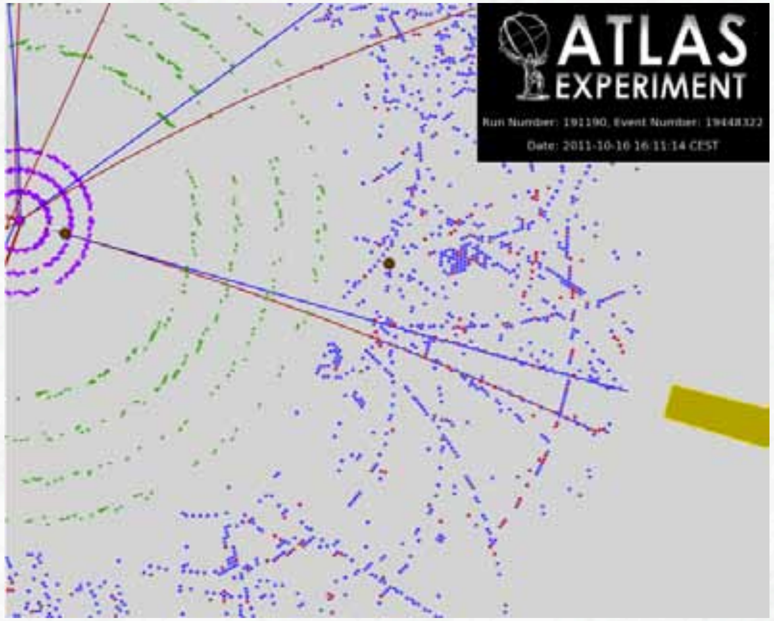
2.1 Measurement of space coordinates

2.2 Basics on momentum measurement

2.3 Silicon semiconductor detectors

2.4 Silicon pixel detectors

2.5 The ATLAS and CMS tracking systems

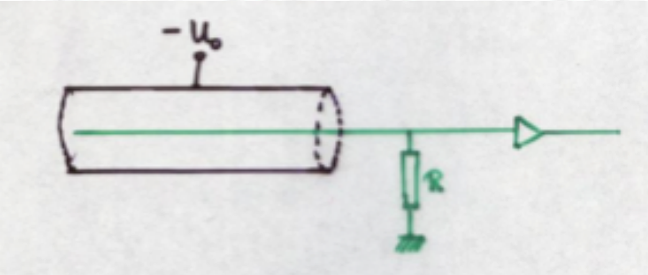


2.1 Measurement of space coordinates

2.1.1 Ionization tubes

- The traditional way to measure space coordinates was based on the measurement of the ionization (signal) in ionization tubes.

- Principle: gas filled tube, high electric field created by a thin wire in the centre (30 μm thickness, anode) and the outer wall (cathode)



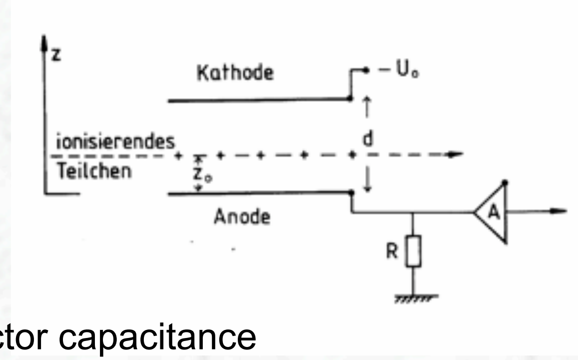
Radius of inner wire: a
Radius of outer wall: b

Electric field: $$E(r) = \frac{U_0}{r} \frac{1}{\ln(b/a)}$$

- Passage of charge particle
  → ionization, drift of liberated electrons and of the positive ions
  → induced signals on anode (the electrode that is connected to an amplifier
  → electrical signal
  (two componenent: fast signal from electrons,
  further slow increase due to slowly drifting ions)
  → use electronic pulse shaping (→ next slide)

- The drift velocity for electrons is larger than for positive ions

Typical values for Ar gas, p_0 (normal pressure), E-field = 500 V/cm:
$v_D^- \sim 5$ cm / μs, $v_D^+ \sim 10^{-3}\ v_D^-$
typical charge collection times: $t^- \sim 1\ \mu$s, $t^+ \sim 1$ ms (too long !)

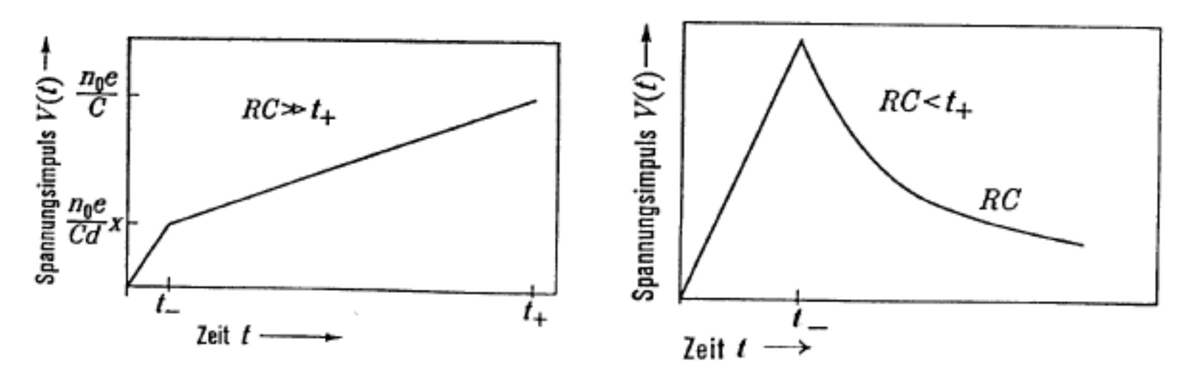


- Differentiate signal via RC-circuit (C = detector capacitance)

- Form of voltage signal:

$$V(t) = \frac{N \cdot e}{C}\left(\frac{1}{t^-} + \frac{1}{t^+}\right) RC\left(1 - e^{-t/RC}\right)$$

C: detector capacitance
N: # e-ions pair produced



→ fast signal

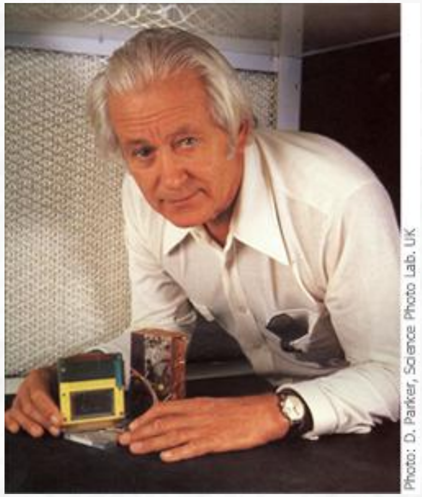
Drawbacks: - not the full signal is collected (signal to noise)
- pulse height depends on the position inside the tube, where the signal is produced

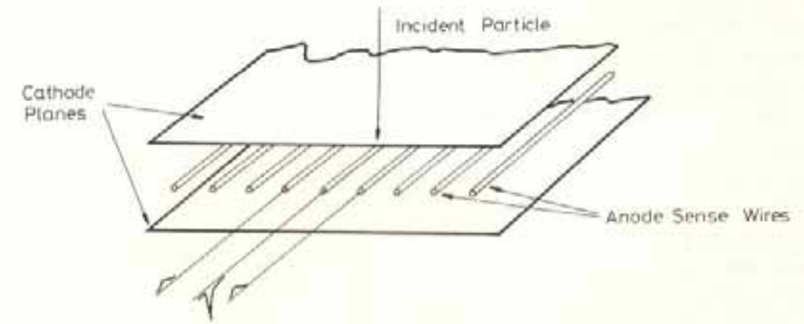
2.1.2 Multiwire proportional chamber

- In order to extract space / coordination information efficiently, Multi-Wire-Proportional Chambers (MWPCs) were used for long time

G. Charpak (Nobel prize,1992)

Principle:
Put many anode wires in parallel, in one volume





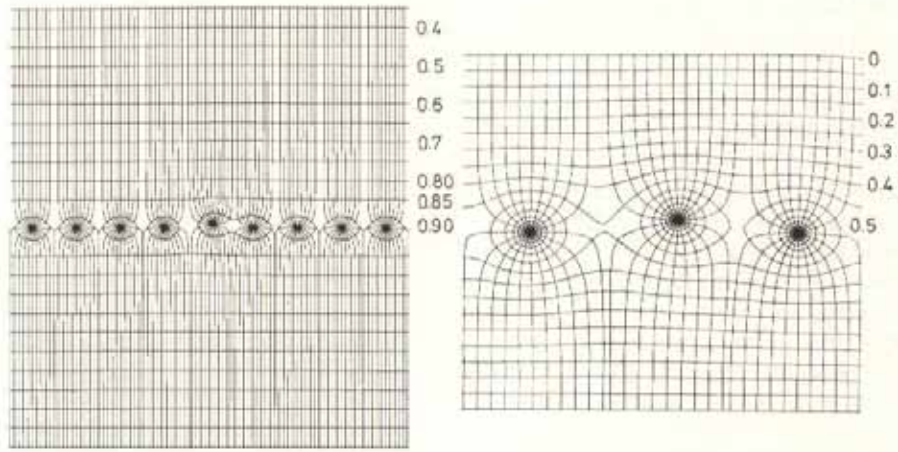
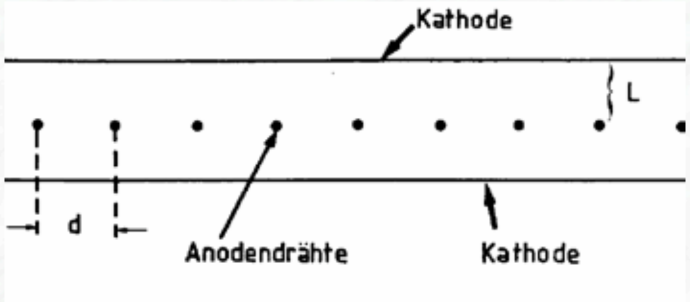


Fig. 6.8. Electric field lines and potentials in a multiwire proportional chamber. The effect of a sli displacement on the field lines is also shown (from *Charpak* et al. [6.16])

- Every wire acts as an independent proportional tube

  → every anode wire is read out separately → space information



- Typical parameters: distance between wires: d = 2 mm
  distance anode-cathode: L = 7-8 mm
  diameter of anode wires: 10 – 30 μm

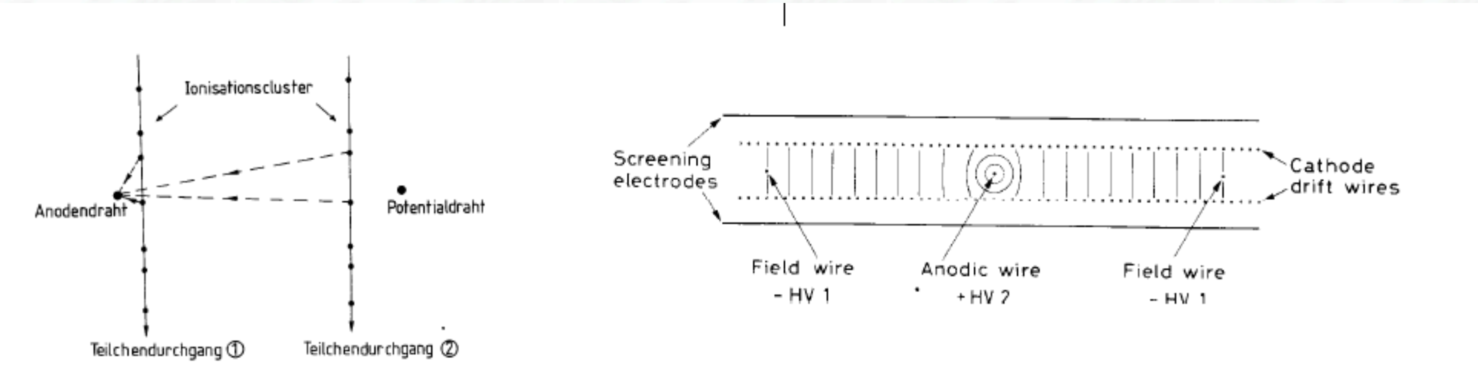
- Achievable coordinate resolution: $\sigma = d / \sqrt{12} \sim 600\ \mu m$

  Can they be used for the LHC ??

  No ! they are too slow, and the precision is not good enough !

2.1.3 Drift chambers

- Measure not only pulse height, but also the time when a signal appears with respect to an external trigger signal



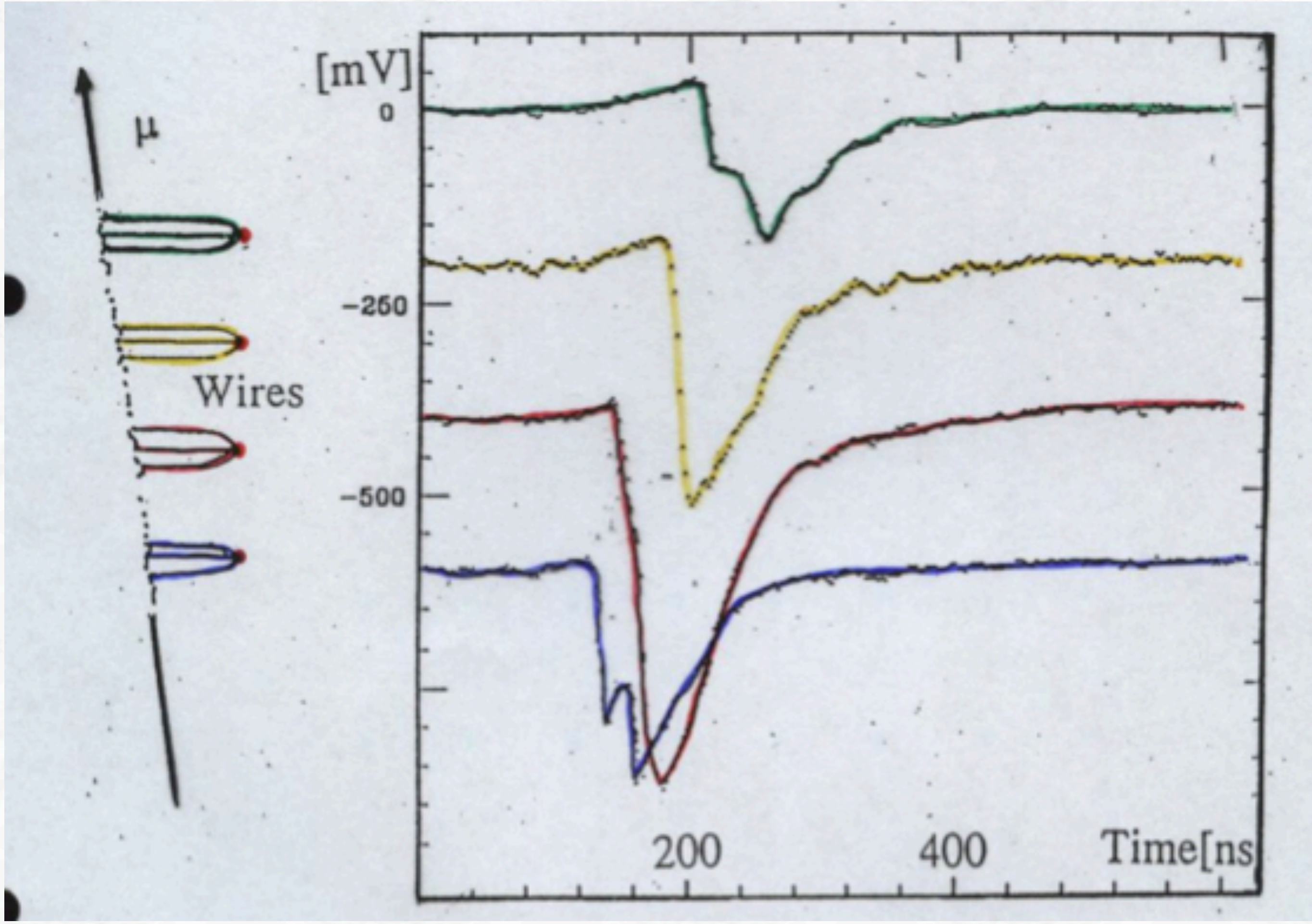
- Position perpendicular to the anode wire can be extracted from a space-drift-time relation
  (has to be known, it is linear, if the drift velocity is constant over the drift volume)

- Typical drift distances: 5 – 10 cm → more economical, less readout channels

- Improved coordinate resolution, typical values: $\sigma \sim$ 50 – 200 μm

  limited by diffusion of the drifting electron clouds, electronics (time measurements)

Measured signals in a drift chamber



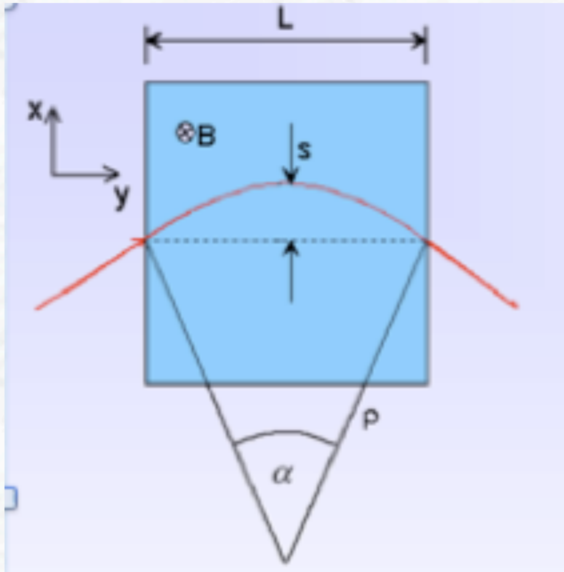
Drift chamber of the CDF experiment (Fermilab)

2.2 Basics on Momentum measurement

- In general the track of a charged particle is measured using several (N) position-sensitive detectors in a magnetic field volume

- Assume that each detector measures the coordinates of the track with a precision of $\sigma(x)$

- The obtainable momentum resolution depends on:
  L (length of the measurement volume)
  B (magnetic field strength)
  σ (position resolution)



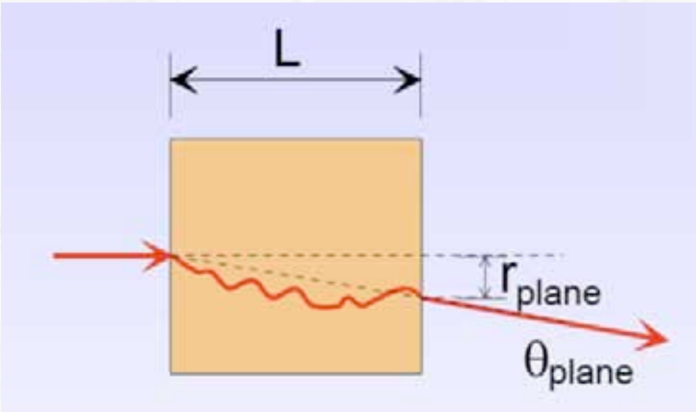
For N equidistant measurements, the momentum resolution is described by the Gluckstern formula (1963):

$$\left.\frac{\sigma(p_T)}{p_T}\right|^{meas.} = \frac{\sigma(x)\cdot p_T}{0.3\cdot BL^2}\sqrt{720/(N+4)} \qquad \text{(for } N \geq \sim 10\text{)}$$

note: $\Delta(p_T) / p_T \sim p_T$ (relative resolution degrades with higher transverse momentum)

Momentum measurement (cont.)

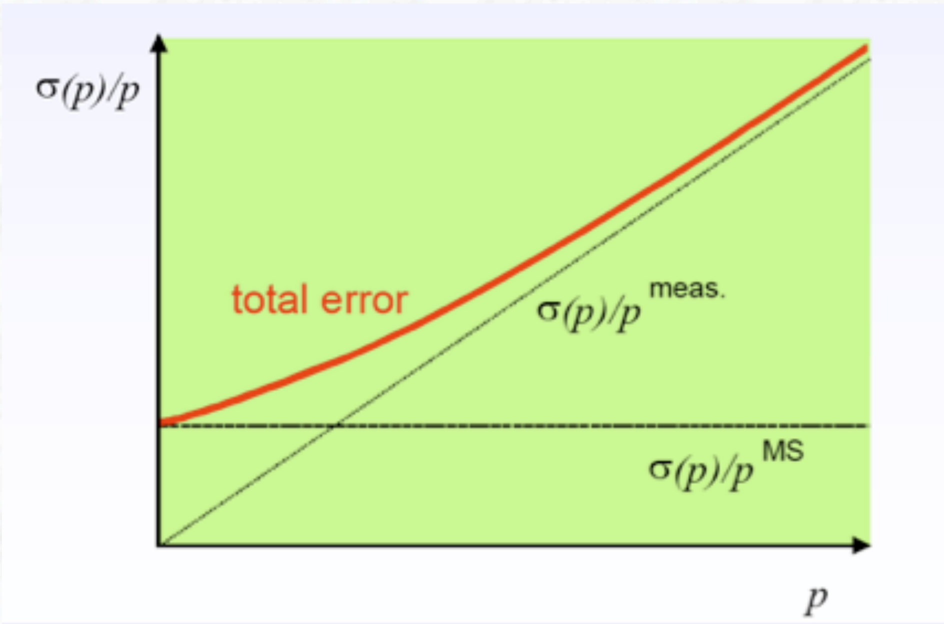
- Degradation of the resolution due to Coulomb multiple scattering (no ionization, elastic scattering on nuclei, change of direction)



$$\theta_0 = \theta_{plane}^{RMS} = \sqrt{\langle \theta_{plane}{}^2 \rangle} = \frac{1}{\sqrt{2}} \theta_{space}^{RMS}$$

$$\theta_0 \propto \frac{1}{p} \sqrt{\frac{L}{X_0}}$$

where X_0 = radiation length of the material (characteristic parameter, see calorimeter section)



$$\left.\frac{\sigma(p)}{p_T}\right|^{MS} = 0.045 \frac{1}{B\sqrt{LX_0}}$$