7. Quantenchromodynamik

7.1 Die Struktur der Wechselwirkung

7.2 Die Symmetriegruppen SU(2) und SU(3)

7.3 Die Feynman-Regeln der QCD

7.4 Matrixelemente und Farbfaktoren

7.5 Die laufende Kopplungskonstante α_s

Die Gell-Mann Matrizen

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

7.3 Die Feynman-Regeln der QCD

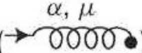
1. *External Lines.* For an external quark with momentum p, spin s, and color c:

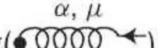
$$\textit{Quark}: \begin{Bmatrix} \text{Incoming } (\rightarrow\bullet): u^{(s)}(p)c \\ \text{Outgoing } (\bullet\rightarrow): \bar{u}^{(s)}(p)c^\dagger \end{Bmatrix}$$

(note that $c^\dagger = \tilde{c}^*$ will be a *row* matrix). For an external *anti*quark:

$$\textit{Antiquark}: \begin{Bmatrix} \text{Incoming } (\leftarrow\bullet): \bar{v}^{(s)}(p)c^\dagger \\ \text{Outgoing } (\bullet\leftarrow): v^{(s)}(p)c \end{Bmatrix}$$

where c represents the color of the corresponding *quark*. For an external gluon of momentum p, polarization ϵ, and color a, include a factor

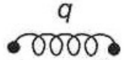
Gluon : Incoming() : $\epsilon_\mu(p)a^\alpha$

Outgoing() : $\epsilon^*_\mu(p)a^{\alpha *}$

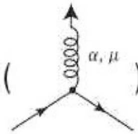
(To avoid confusion it is helpful to indicate on the diagram the indices – space-time and color – you are using for each gluon.)

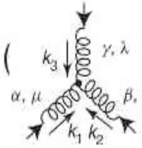
2. *Propagators.* Each internal line contributes a factor

Quarks and antiquarks: $(\bullet\xrightarrow{q}\bullet)$: $\dfrac{i(\not q + mc)}{q^2 - m^2c^2}$

Gluons: () : $\dfrac{-ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}$

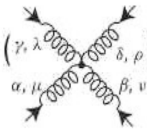
3. *Vertices.* Each vertex introduces a factor

Quark-gluon: () : $\dfrac{-ig_s}{2}\lambda^\alpha\gamma^\mu$

Three gluon: () :

$$-g_s f^{\alpha\beta\gamma}[g_{\mu\nu}(k_1 - k_2)_\lambda + g_{\nu\lambda}(k_2 - k_3)_\mu + g_{\lambda\mu}(k_3 - k_1)_\nu]$$

Here the gluon momenta (k_1, k_2, k_3) are assumed to point *into* the vertex; if any point *outward* in your diagram, change their signs.

Four gluon: () :

$$-ig_s^2[f^{\alpha\beta\eta}f^{\gamma\delta\eta}(g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}) + f^{\alpha\delta\eta}f^{\beta\gamma\eta}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho})$$
$$+ f^{\alpha\gamma\eta}f^{\delta\beta\eta}(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\lambda\rho})]$$

(summation over η implied).