Exercises for Advanced Particle Physics - Winter term 2013/14

Exercise sheet No. V

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The solutions have to be returned to mail box no. 1
in the foyer of the Gustav-Mie-House before **Monday, December 2nd, 12:00h**.

Elementary processes, transition amplitudes and cross sections

In the previous exercise sheet, the complete calculation of the $e^+e^- \to \mu^+\mu^-$ cross section was performed from first principles. The following exercises aim to build an intuition of the main kinematic properties of three typical Feynman diagrams. An introduction to the PHYTIA Monte Carlo program is also proposed, to be able to start the simulation of processes in the next weeks.

Exercise No. 1: Kinematic dependence of typical amplitudes (5 points)

We consider a (2) $\to$ (2) process, where the initial (final) 4-momentum are p_1, p_2 (k_1, k_2). We recall that the Mandelstam variables are defined by

$$s = (p_1 + p_2)^2 = (k_1 + k_2)^2 \quad (1)$$

$$t = (p_1 - k_1)^2 = (p_2 - k_2)^2 \quad (2)$$

$$u = (p_1 - k_2)^2 = (p_2 - k_1)^2. \quad (3)$$

1. Write the three Feynman diagrams associated to these 3 variables. We call these amplitudes the $s-$channel, the $t-$channel and the $u-$channel. Hint: the squared momentum carried by the virtual photon must be s, t and u.

2. Show that, in the ultra-relativistic limit, the matrix element of the $e^+e^- \to \mu^+\mu^-$ computed last week (noted $\mathcal{M}_s$)

$$\overline{|\mathcal{M}_s|^2} = \frac{8e^4}{(p_1+p_2)^4}\left[(p_2\cdot p_4)(p_1\cdot p_3) + (p_2\cdot p_3)(p_1\cdot p_4) + m_\mu^2(p_1\cdot p_2)\right] \quad (4)$$

   can be expressed as

$$\overline{|\mathcal{M}_s|^2} = 2e^4\left(\frac{u^2+t^2}{s^2}\right) \quad (5)$$

3. Using the diagrams of the question (1) and the crossing symmetry, show that

$$\overline{|\mathcal{M}_t|^2} = 2e^4\left(\frac{s^2+u^2}{t^2}\right) \quad , \quad \overline{|\mathcal{M}_u|^2} = 2e^4\left(\frac{s^2+t^2}{u^2}\right) \quad (6)$$

4. Study of the *Møller scattering*: $e^-e^- \to e^-e^-$. What are the possible amplitudes for this process ? By neglecting the interference between the two amplitudes, discuss the dependence of the total squared amplitude with the scattering angle θ.

5. Study of the *Bhabha scattering*: $e^+e^- \to e^+e^-$. What are the possible amplitudes for this process ? By neglecting the interference between the two amplitudes, discuss the dependence of the total squared amplitude with the scattering angle θ. Compare the result to Møller scattering.

6. We can show that the interference term is $2s^2/tu$ for the Møller scattering and $2u^2/ts$ for the Bhabha scattering. Does it change the main conclusions above ?

Exercise No. 2: Introduction to PHYTIA **(2 points)**

PHYTIA is a program allowing to simulate reactions for a given theory. We propose in this exercise to get familiar with it, from a technical point of view. Later, we will use it to actually simulate some reactions studied in the lecture.

1. Download the PHYTIA 8 program on http://home.thep.lu.se/ torbjorn/Pythia.html. Follow the instruction and compile the program.
2. Run the example main01.cc. Modify the example to change the beam energy to 14 TeV.

Experimental tests of QED

Exercise No. 3: Cross section behaviour at the threshold **(3 points)**

We have shown that the behaviour of the cross section at the energy threshold is determined by the structure of the QED interaction. We propose to compare experimental measurements and theoretical predication for the QED production of a fermion pair

$$\sigma_{\text{total}} = \frac{\pi\alpha^2}{3E^2}\sqrt{1-\frac{m_f^2}{E^2}}\left(1+\frac{m_f^2}{2E^2}\right) \tag{7}$$

E_{cm} (GeV)	$\sigma(e^+e^- \to \tau^+\tau^-)/\sigma(e^+e^- \to \mu^+\mu^-)$
3.45	0.000 ± 0.003
3.52	0.001 ± 0.006
3.57	0.015 ± 0.006
3.62	0.028 ± 0.012
3.68	0.067 ± 0.010
3.72	0.064 ± 0.016
3.74	0.055 ± 0.018
3.77	0.088 ± 0.007
3.85	0.061 ± 0.011
3.95	0.106 ± 0.018
4.05	0.110 ± 0.013
4.16	0.130 ± 0.017
4.25	0.137 ± 0.020
4.38	0.145 ± 0.013

Table 1: Ratio $\sigma(e^+e^- \to \tau^+\tau^-)/\sigma(e^+e^- \to \mu^+\mu^-)$ versus the centre-of-mass energy, as measured in the DELCO experiment.

- Use a computer program to plot the points given in Table 1
- Perform a fit of the data points using the theoretical prediction given in equation 7
- Deduce the value of m_τ

Hint: for some experimental reason, the overall normalization of the ratio can disagree with the prediction, but the dependence with E_{cm} contains the information.