Exercises for Advanced Particle Physics - Winter term 2013/14

Exercise sheet No. X

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The solutions have to be returned to mail box no. 1
in the foyer of the Gustav-Mie-House before **Monday, January 27th, 12:00h**.

Forward-backward asymmetry in the electroweak interaction

The structure of the electroweak interaction leads to specific features that can be experimentally probed by analysing high-energy collisions or even precision measurements in atomic physics. This exercise aims to illustrate how the measurements of differential cross sections allow to probe the particular structure of the electroweak interaction. We consider the $e^+e^- \to \mu^+\mu^-$ process in collisions at a centre-of-mass energy $\sqrt{s}$. The differential cross section, computed at the leading order in perturbation theory, is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{\pi\alpha^2}{2s}\left(F_\gamma(\cos\theta) + F_Z(\cos\theta)\frac{s^2}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2} + F_{Z\gamma}(\cos\theta)\frac{s(s-m_Z^2)}{(s-m_Z^2)^2+m_Z^2\Gamma_Z^2}\right) \quad (1)$$

$$F_\gamma(\cos\theta) = 1+\cos^2\theta \quad (2)$$

$$F_Z(\cos\theta) = \frac{1}{16\,\sin^2\theta_W\cos^2\theta_W}\left(({c_V}^2+{c_A}^2)^2\,(1+\cos^2\theta)+8c_A^2c_V^2\cos\theta\right) \quad (3)$$

$$F_{Z\gamma}(\cos\theta) = \frac{1}{4\,\sin^2\theta_W\cos^2\theta_W}\left(2c_V^2(1+\cos^2\theta)+4c_A^2\cos\theta\right) \quad (4)$$

The axial-vector (c_A) and vector (c_V) coupling constants for charged leptons are predicted by the theory to be:

$$c_A = -\frac{1}{2}\,, \quad c_V = -\frac{1}{2}+2\,\sin^2\theta_W \quad (5)$$

1. Plot the differential cross section for the three terms in equation (1) for $\sqrt{s} = m_Z - 5$ GeV, $\sqrt{s} = m_Z$ and $\sqrt{s} = m_Z + 5$ GeV. Use $\sin^2\theta_W = 0.23$, $m_Z = 91.2$ GeV, $\Gamma_Z = 2.5$ GeV). Comment on the following points:
   - Where does the forward-backward asymmetry come from?
   - Where does the dependence with $\sqrt{s}$ come from?
2. More quantitatively, we define the forward-backward asymmetry A_{FB} by

$$A_{FB} = \frac{\sigma_F-\sigma_B}{\sigma_F+\sigma_B}\,, \quad \sigma_{F(B)} = \int_{0(-1)}^{1(0)}\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\mathrm{d}\cos\theta \quad (6)$$

   - Compute $\sigma_{F(B)}$ for each term of equation (1)
   - Compute and plot A_{FB} as a function of c_V, c_A and the centre-of-mass energy.

3. The process $e^+e^- \to \mu^+\mu^-$ was generated for the three centre-of-mass energies given in Part (1.). The corresponding root files are accessible at

   `http://rmadar.web.cern.ch/rmadar/Teaching/Pyhtia/EW/`

   By plotting the $\cos\theta$ distribution, measure A_{FB} for the three centre-of-mass energies. Show that the result is compatible with equation (5) and deduce a measured value of $\sin^2\theta_W$. Hint: a fit of the theoretical formula can be performed by leaving c_A and c_V as free parameters.