6. Physics of W and Z bosons

6.1 The theory of electroweak interactions (a short repetition)

6.2 Summary of precision tests at LEP

6.3 W and Z boson production in hadron colliders

6.4 Test of QCD in W/Z (+jet) production

6.5 W mass measurement

Weak Isospin and Hypercharge Quantum

Lepton	T	T^3	Q	Y
ν_e	$\frac{1}{2}$	$\frac{1}{2}$	0	-1
e_L^-	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1
e_R^-	0	0	-1	-2

Numbers of Leptons and Quarks

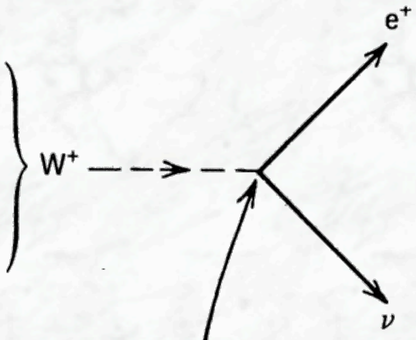
Quark	T	T^3	Q	Y
u_L	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{1}{3}$
d_L	$\frac{1}{2}$	-$\frac{1}{2}$	-$\frac{1}{3}$	$\frac{1}{3}$
u_R	0	0	$\frac{2}{3}$	$\frac{4}{3}$
d_R	0	0	-$\frac{1}{3}$	-$\frac{2}{3}$

Important Milestones towards Electroweak Unification

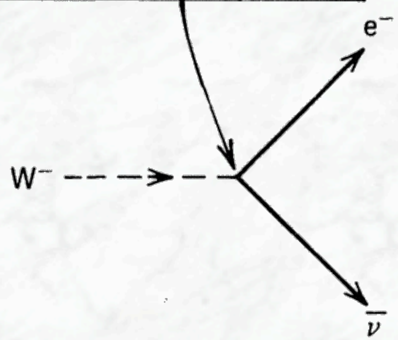
1961	S. Glashow proposes an electroweak gauge theory, Introduction of massive $W^{\pm}$ and Z^{0} bosons, to explain the large difference in strength of electromagnetic and weak interactions. Key question: how acquire W and Z bosons mass?
1964	R. Brout, F. Englert and P. Higgs demonstrate that mass terms for gauge bosons can be introduced in local gauge invariant theories via spontaneous symmetry breaking
1967	S. Weinberg and A. Salam use Brout-Englert-Higgs mechanism to introduce mass terms for W and Z bosons in Glashow's theory → GSW theory (Glashow, Salam, Weinberg) → mass terms for W, Z bosons, γ remains massless → Higgs particle (see chapter 7)
1973	G. t'Hooft and M. Veltman show that GSW theory is renormalizable
1979	Nobel price for S. Glashow, A. Salam and S. Weinberg
1983	Experimental discovery of the W and Z bosons by UA1 and UA2 experiments at the CERN ppbar collider ($\sqrt{s}$ = 540 GeV)
1990-2000	Precise test of the electroweak theory at LEP
1999	Nobel price for G. t'Hooft and M. Veltman
2012	Discovery of a Higgs particle by the ATLAS and CMS experiments at the LHC

W and Z vertex factors

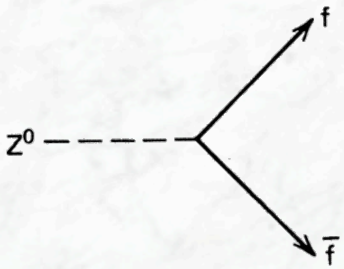
$$-i\frac{g}{\sqrt{2}}\left(\bar{\chi}_L\gamma^\mu\tau_+\chi_L\right)W_\mu^+$$
$$= -i\frac{g}{\sqrt{2}}\left(\bar{\nu}_L\gamma^\mu e_L\right)W_\mu^+$$



$$-i\frac{g}{\sqrt{2}}\gamma^\mu\tfrac{1}{2}\left(1-\gamma^5\right)$$



$$-i\frac{g}{\sqrt{2}}\left(\bar{\chi}_L\gamma^\mu\tau_-\chi_L\right)W_\mu^-$$
$$= -i\frac{g}{\sqrt{2}}\left(\bar{e}_L\gamma^\mu\nu_L\right)W_\mu^-$$

⇨ 

$$-i\frac{g}{\cos\theta_W}\gamma^\mu\tfrac{1}{2}\left(c_V^f-c_A^f\gamma^5\right).$$

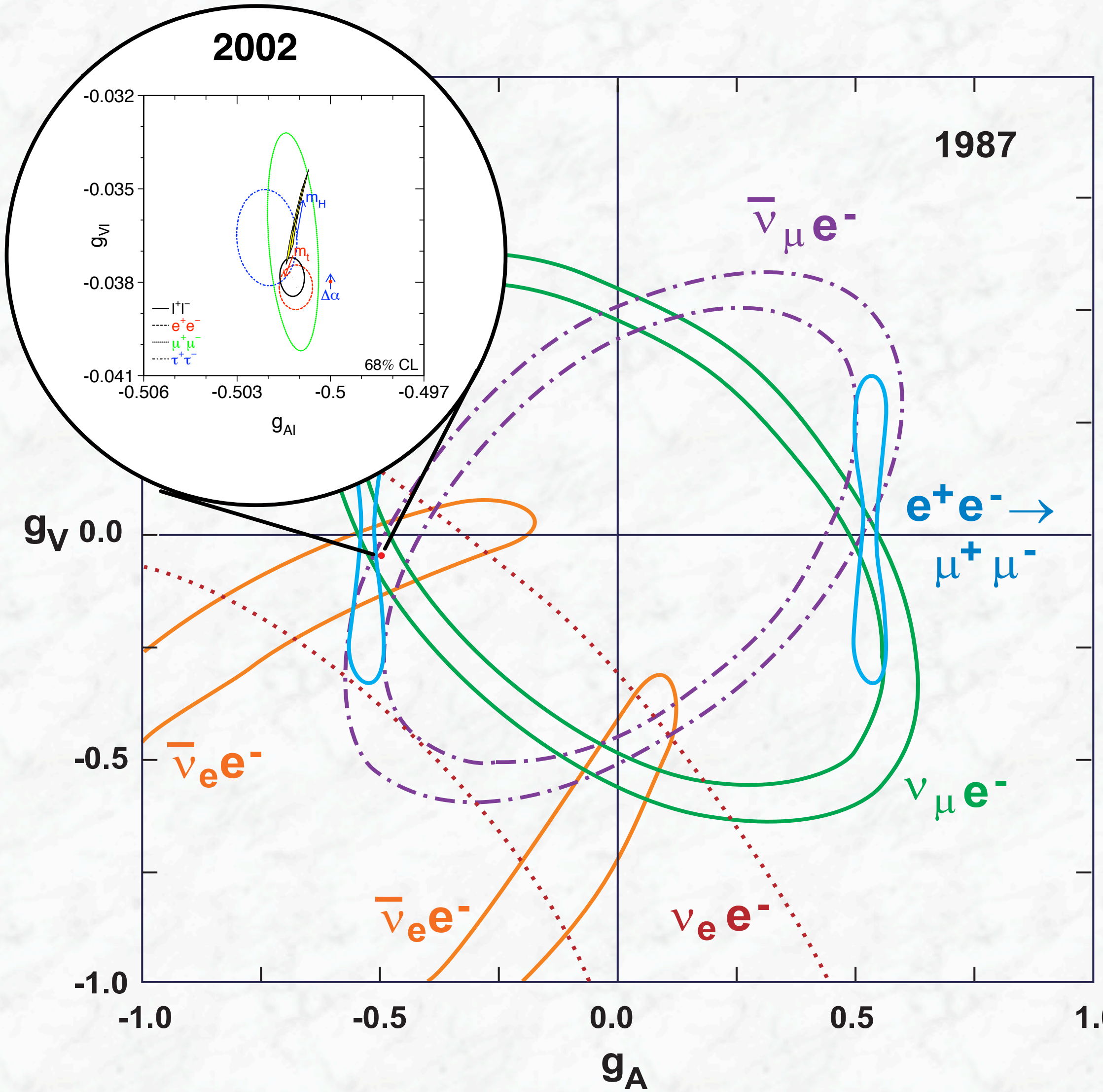
The Z → ff vertex factors in the Standard Model

($\sin^2 \theta_W$ is assumed to be 0.234)

f	$\mathbf{Q}_f$	$\mathbf{c}_A^f$	$\mathbf{c}_V^f$
$\nu_e, \nu_\mu, \ldots$	0	$\frac{1}{2}$	$\frac{1}{2}$
$e \quad , \mu \quad , \ldots$	-1	$-\frac{1}{2}$	$-\frac{1}{2} + 2\sin^2\theta_W \quad 0.03$
$u, c, \ldots$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{2} \quad \frac{4}{3}\sin^2\theta_W \quad 0.19$
$d, s, \ldots$	$-\frac{1}{3}$	$-\frac{1}{2}$	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W \quad 0.34$

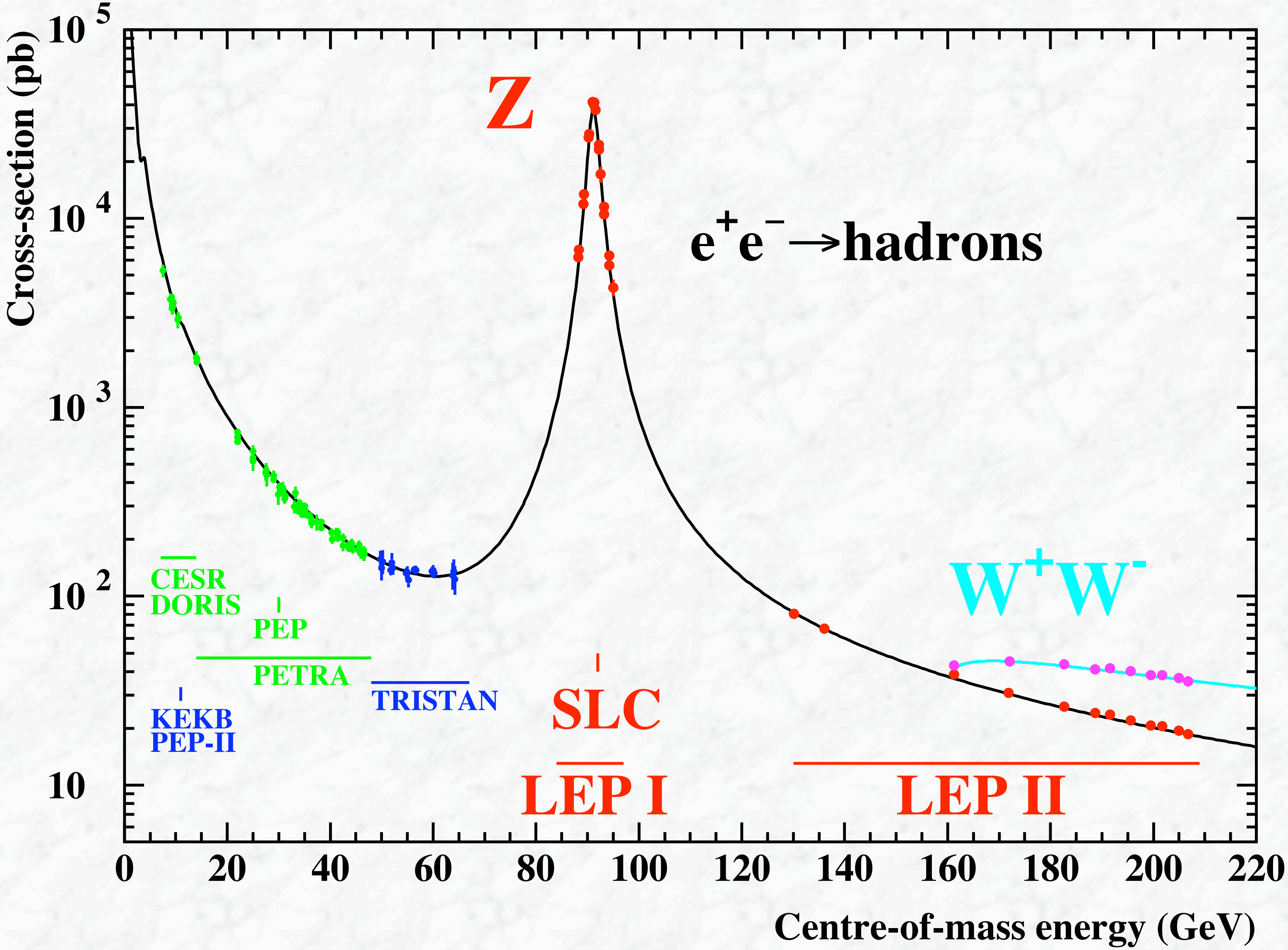
6.2 Summary of electroweak precision tests at LEP

- Results of 30 years of experimental and theoretical progress
- The electroweak theory is tested at the level of 10^{-4}




LEP am CERN / Genf
ALEPH
L3
LEP / LHC
SPS
OPAL
DELPHI
e+e- -Beschleuniger, 27 km Umfang
Schwerpunktsenergie: LEP-I (1989-1995) 91 GeV
LEP-II (1996-2000) → 208 GeV
4 Experimente: ALEPH, DELPHI, L3, OPAL

Cross sections for W and Z boson production



Precision tests of the Z sector

Tests of the W sector

Cross section for $e^+e^- \rightarrow \mu^+\mu^-$ at LEP I

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[F_\gamma(\cos\theta) + F_{\gamma Z}(\cos\theta)\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} + F_Z(\cos\theta)\frac{s^2}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}\right]$$

γ γ/Z interference Z

γ/Z interference vanishes at $\sqrt{s} \approx M_Z$

$$F_\gamma(\cos\theta) = Q_e^2 Q_\mu^2 (1+\cos^2\theta) = (1+\cos^2\theta)$$

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W\cos^2\theta_W}[2g_V^e g_V^\mu(1+\cos^2\theta) + 4g_A^e g_A^\mu \cos\theta]$$

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W\cos^4\theta_W}[(g_V^{e^2} + g_A^{e^2})(g_V^{\mu^2} + g_A^{\mu^2})(1+\cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta]$$

$\alpha=\alpha(m_Z)$: running el.magnetic coupling [$\alpha(M_Z) = \alpha/(1-\Delta\alpha)$ mit $\Delta\alpha\approx 0.06$]

$g_V, g_A = c_V, c_A$: effective coupling constants (vector and axial vector)

Cross section for $e^+e^- \rightarrow f\bar{f}$ at LEP I

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[F_\gamma(\cos\theta) + F_{\gamma Z}(\cos\theta)\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} + F_Z(\cos\theta)\frac{s^2}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}\right]$$

γ | γ/Z interference | Z

γ/Z interference vanishes at $\sqrt{s} \approx M_Z$

x N_C^f
number of colour degrees of freedom for für fermion f

x $(1+\delta_{QCD})$
QCD correction term

$$F_\gamma(\cos\theta) = Q_e^2 Q_f^2 (1+\cos^2\theta) = (1+\cos^2\theta)$$

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_f}{4\sin^2\theta_W\cos^2\theta_W}\left[2g_V^e g_V^\mu(1+\cos^2\theta) + 4g_A^e g_A^f\cos\theta\right]$$

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W\cos^4\theta_W}\left[(g_V^{e^2}+g_A^{e^2})(g_V^{f^2}+g_A^{f^2})(1+\cos^2\theta) + 8g_V^e g_A^e g_V^f g_A^f\cos\theta\right]$$

Cross section for $e^+e^- \rightarrow f\bar{f}$ on resonance ($\sqrt{s} = m_Z$)

- On resonance, $\sqrt{s} = m_Z$:
  - γ^*/Z interference terms vanishes
  - γ term contributes ~1%
  - Z contribution dominates !

- Contribution of the γ^*/Z interference term at $s = (M_Z - 3\ \text{GeV})^2$: ~0.2%

Total cross section for $e^+e^- \rightarrow \mu^+\mu^-$ (integration over cos θ)

$$\sigma_{\text{tot}} \approx \sigma_Z = \frac{4\pi}{3s} \frac{\alpha^2}{16 \sin^4\theta_W \cos^4\theta_W} \cdot [(g_V^e)^2 + (g_A^e)^2][(g_V^\mu)^2 + (g_A^\mu)^2] \cdot \frac{s^2}{(s - M_Z^2)^2 + (M_Z\Gamma_Z)^2}$$

$$\sigma_Z(\sqrt{s} = M_Z) = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

Peak cross section

$$\Gamma_f = \frac{\alpha M_Z}{12 \sin^2\theta_W \cos^2\theta_W} \cdot [(g_V^f)^2 + (g_A^f)^2]$$

Partial width

$$\Gamma_Z = \sum_i \Gamma_i$$

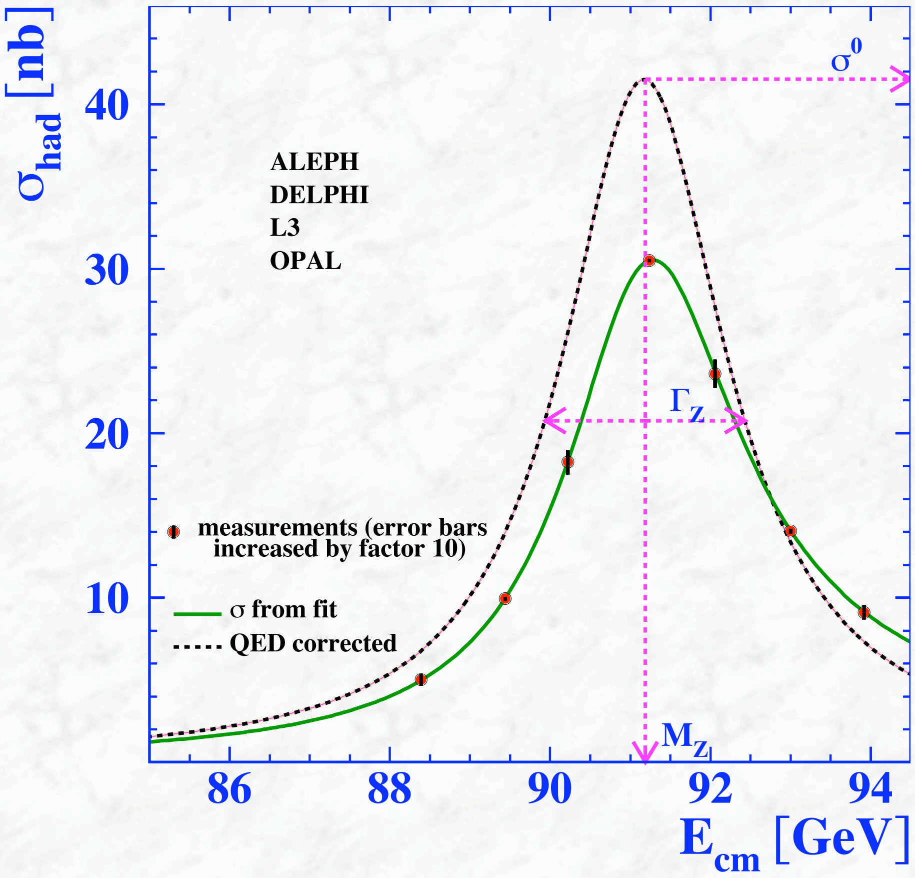
Total width

From the energy dependence of the total cross section (for various fermions f) the parameters

M_Z, Γ_Z, Γ_f

can be determined.

Measurement of the Z line-shape



Line shape (resonance curve):

$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s-M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Peak: $\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$

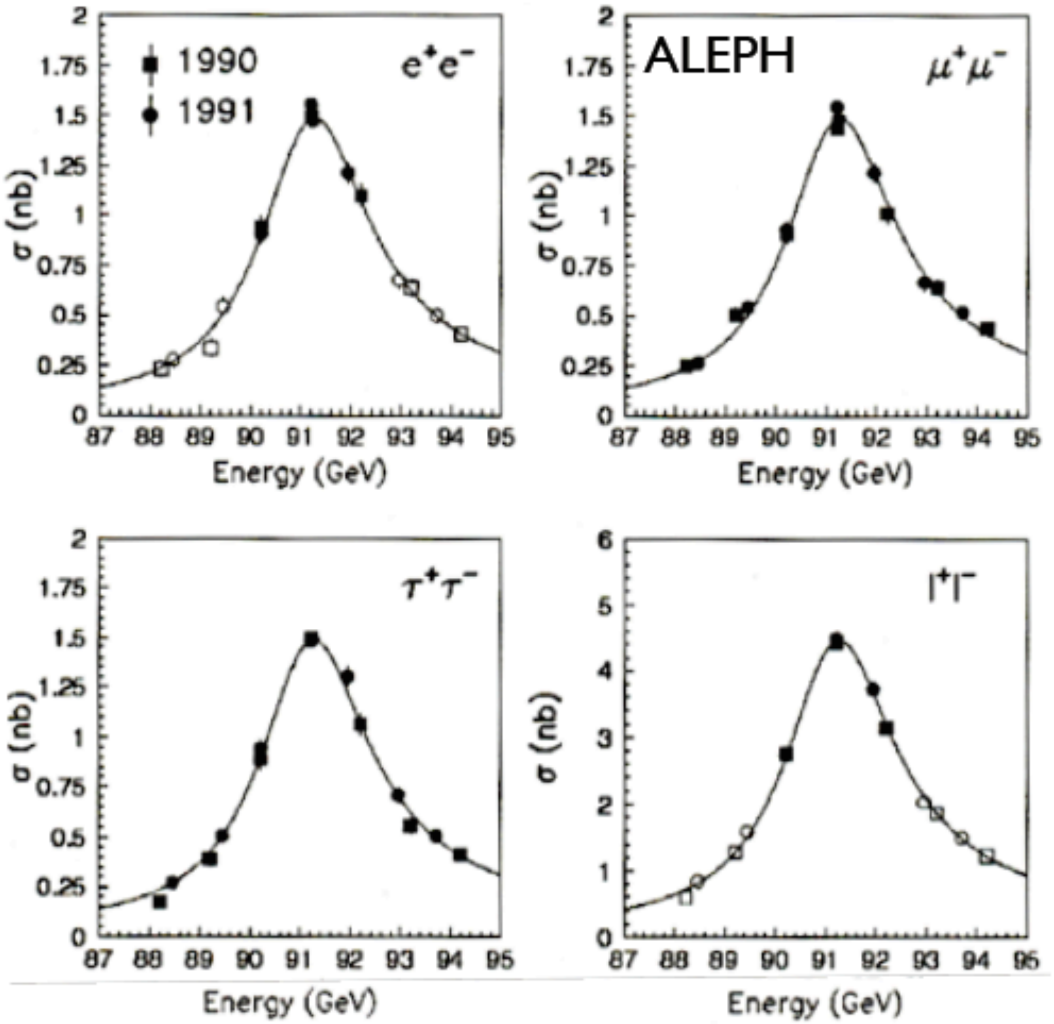
- Position of maximum $\rightarrow$ M_Z
- Full width at half maximum $\rightarrow$ Γ_Z
- Peak cross section σ_0 $\rightarrow$ $\Gamma_e \Gamma_\mu$

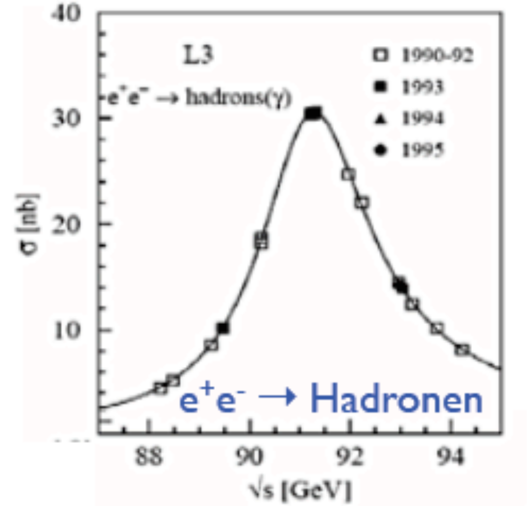
Radiative corrections (photon radiation) important

 with ISR (initial state radiation)

 without ISR

Measurement of the Z line-shape (cont.)





Quark-Flavor i.a. nicht exp. trennbar
(Ausnahme: c,b → Lebendsdauer)
⇒ had. Breite: $\Gamma_{had} = \Gamma_u+\Gamma_d+\Gamma_s+\Gamma_c+\Gamma_b$

Messe Verhältnisse der Pol-WQ:

$$R_l^0 \equiv \frac{\Gamma_{had}}{\Gamma_{ll}} \qquad l = e, \mu, \tau$$

$$R_q^0 \equiv \frac{\Gamma_{qq}}{\Gamma_{had}} \qquad q = b, c$$

- Keine Unterschiede für verschiedene Leptonarten ⇒ Leptonuniversalität
- Form der Resonanzenkurve für alle Endzustände gleich (gleicher Propagator!)

Results on Z line-shape parameters

M_Z = 91.1876 ± 0.0021 GeV   23 ppm (*)

Γ_Z	= 2.4952 ± 0.0023	GeV
Γ_{had}	= 1.7458 ± 0.0027	GeV
Γ_e	= 0.08392 ± 0.00012	GeV
Γ_μ	= 0.08399 ± 0.00018	GeV
Γ_τ	= 0.08408 ± 0.00022	GeV

3 lepton flavours treated independently

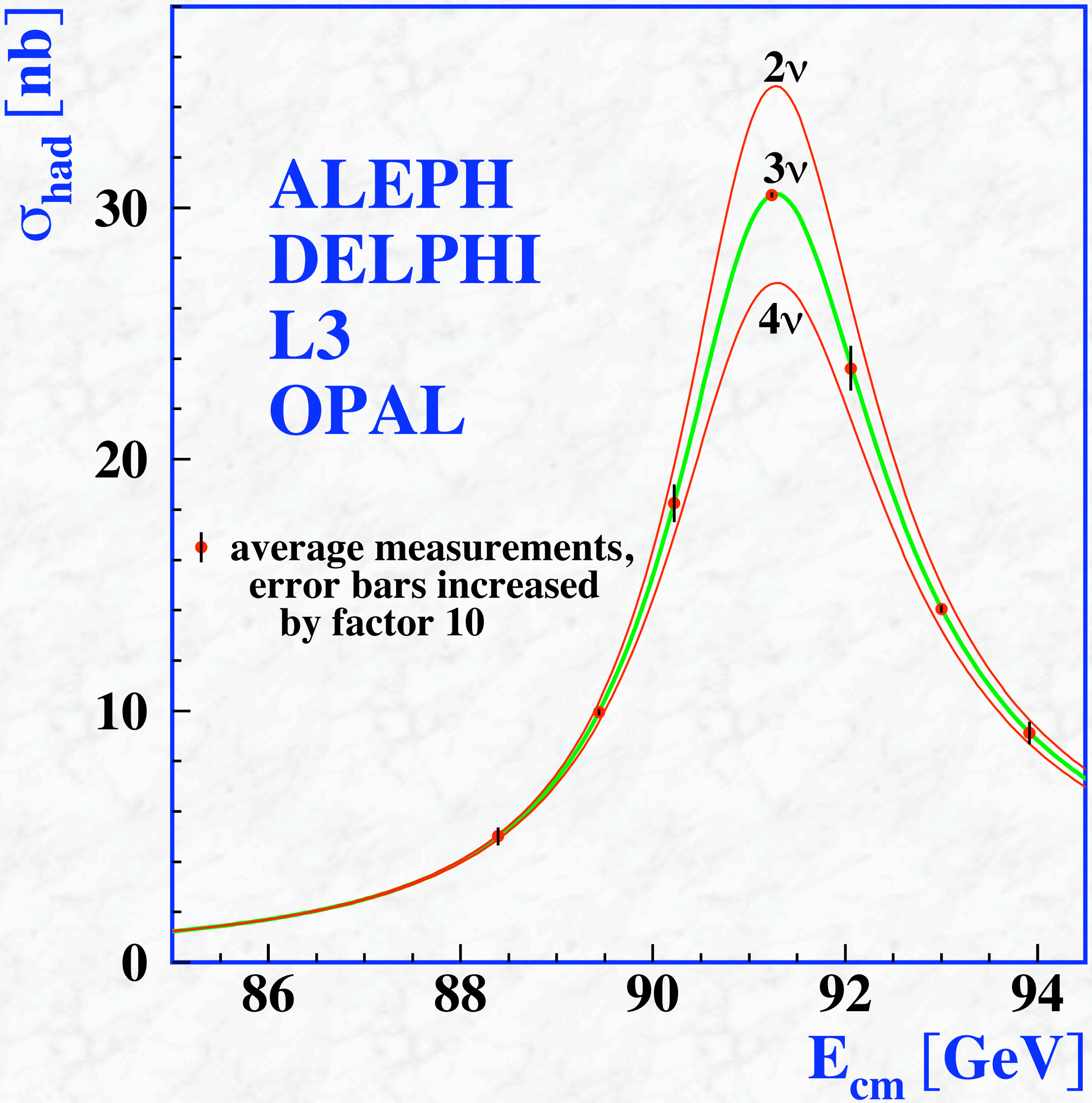
Test of lepton universality

Γ_Z	= 2.4952 ± 0.0023	GeV
Γ_{had}	= 1.7444 ± 0.0022	GeV
Γ_e	= 0.083985 ± 0.000086	GeV

lepton universality assumed:
$\Gamma_e = \Gamma_\mu = \Gamma_\tau$

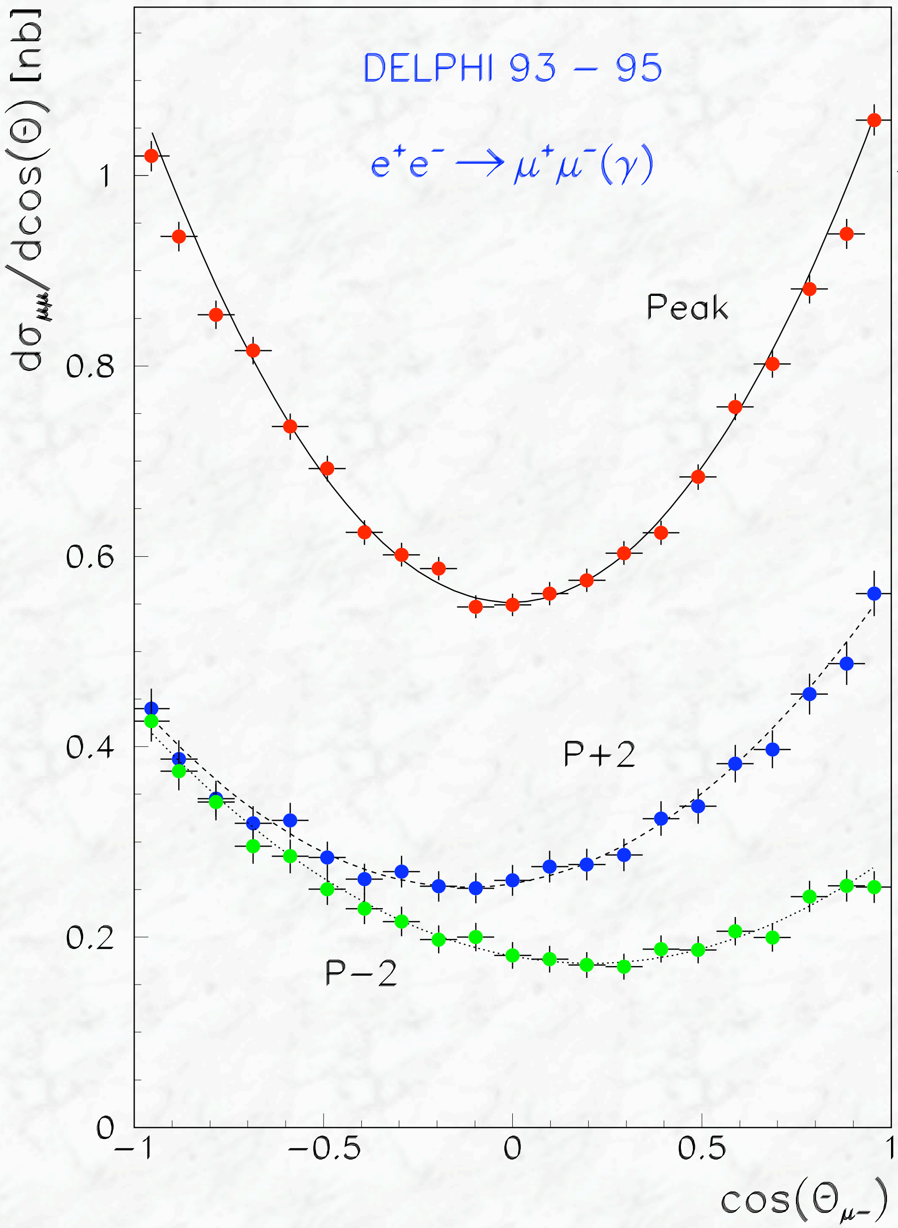
*) Uncertainty on LEP energy measurement: ± 1.7 MeV (19 ppm)

Number of neutrinos



$N_\nu = 2.9840 \pm 0.0082$

Forward-backward asymmetries



$$F_\gamma(\cos\theta) = Q_e^2 Q_\mu^2 (1+\cos^2\theta) = (1+\cos^2\theta)$$

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W \cos^2\theta_W}[2g_V^e g_V^\mu(1+\cos^2\theta) + 4g_A^e g_A^\mu \cos\theta]$$

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W \cos^4\theta_W}[(g_V^{e^2} + g_A^{e^2})(g_V^{\mu^2} + g_A^{\mu^2})(1+\cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta]$$

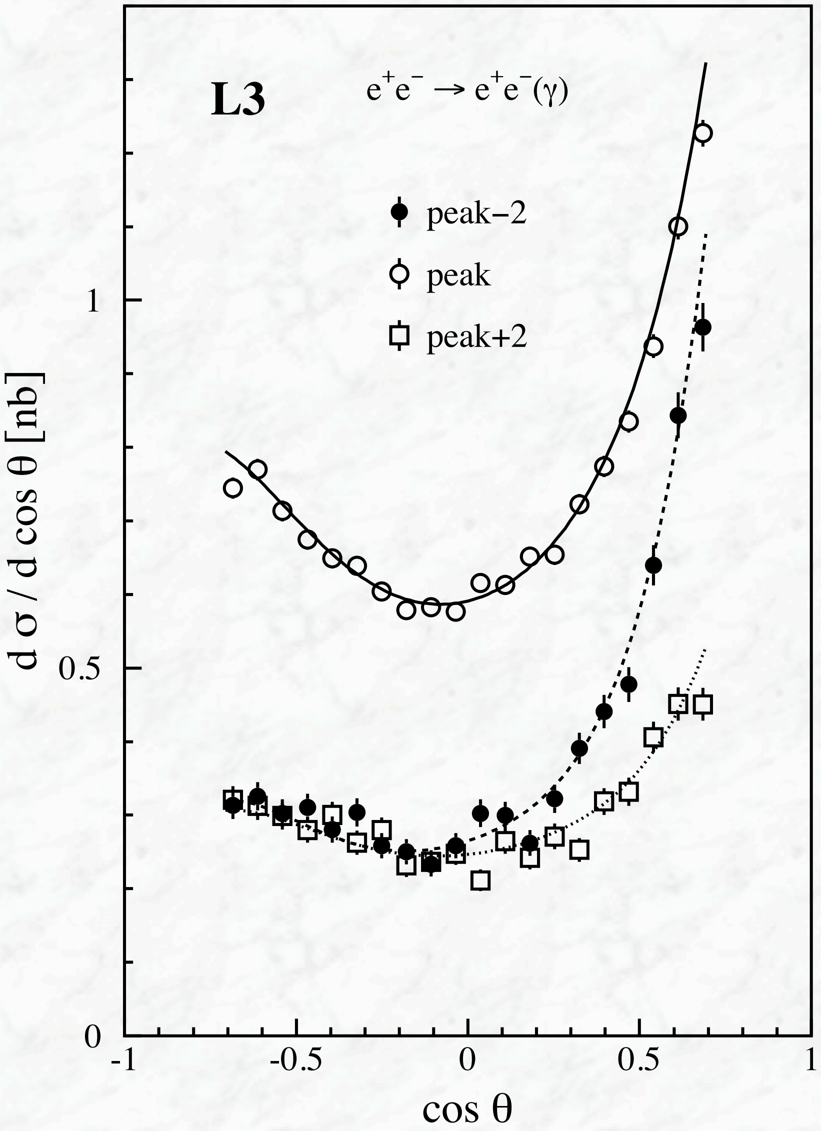
Terms ∝ cosθ in dσ/dcosθ
→ asymmetry

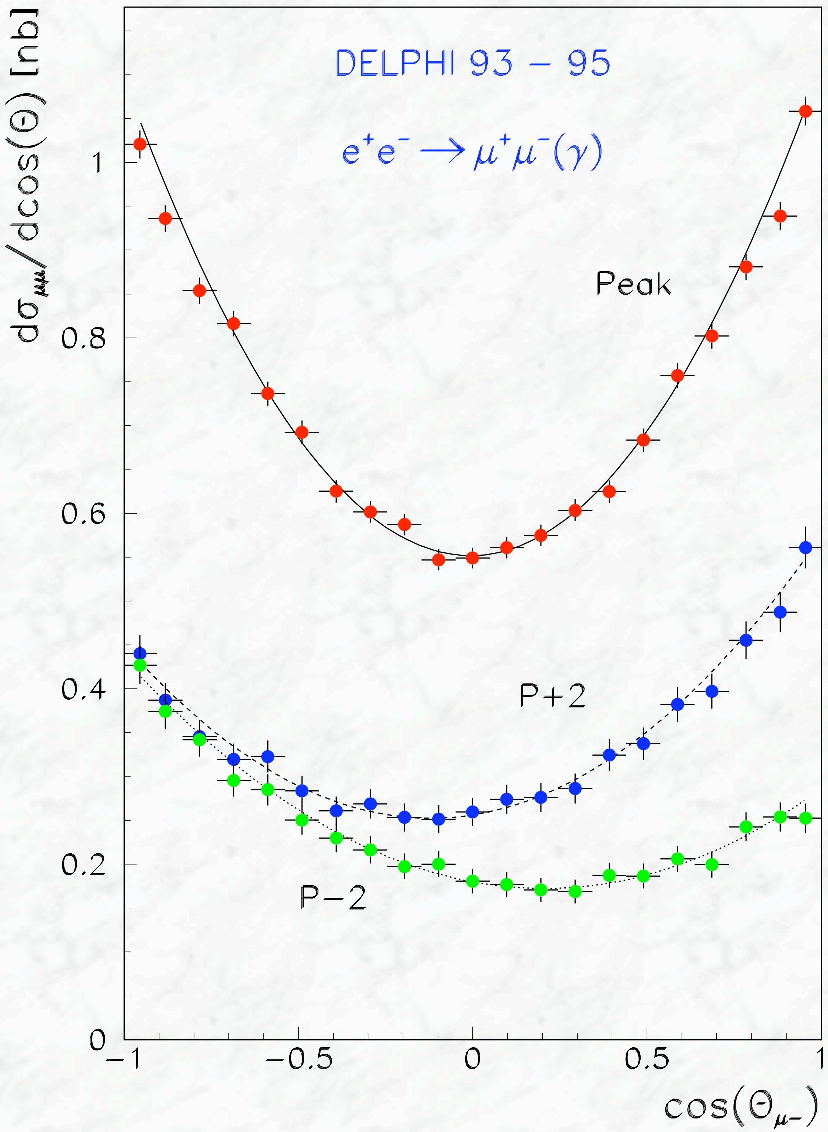
$$\sigma_{F(B)} = \int_{0(-1)}^{1(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta$$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

Forward-backward asymmetries

-comparison between ee and μμ final states-





Forward-backward asymmetries and fermion couplings

- Asymmetry at the Z pole (no interference) is small

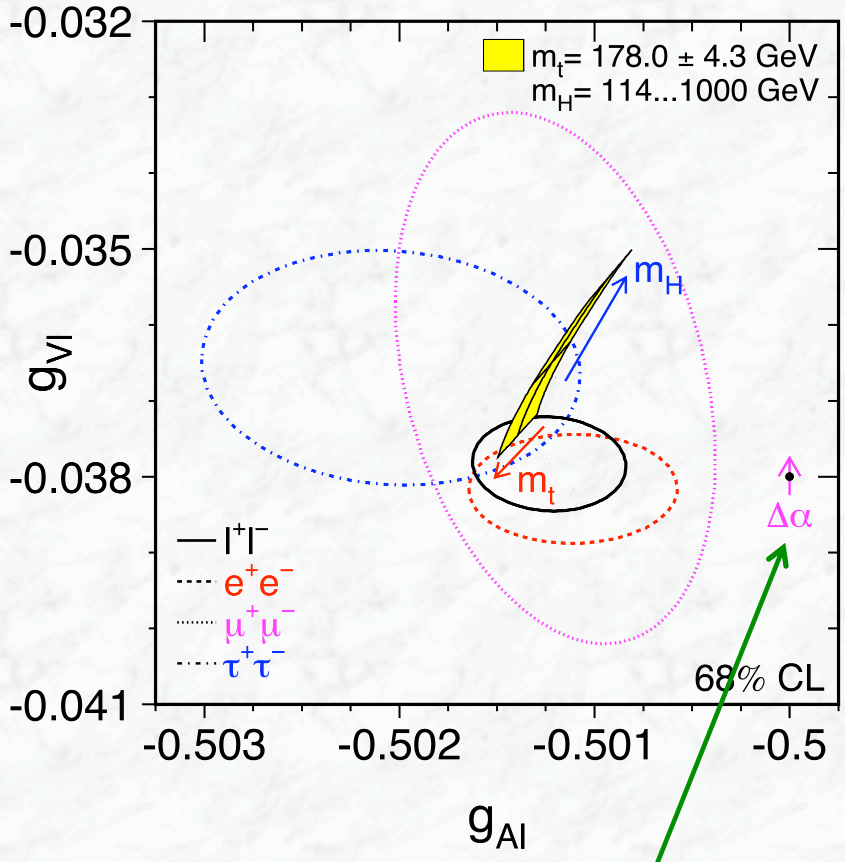
$$A_{\mathrm{FB}} \sim g_{\mathrm{A}}^{e} g_{\mathrm{V}}^{e} g_{\mathrm{A}}^{f} g_{\mathrm{V}}^{f}$$

since g_V^f is small (in particular for leptons)

- For off-resonance points, the interference term dominates and gives larger contributions

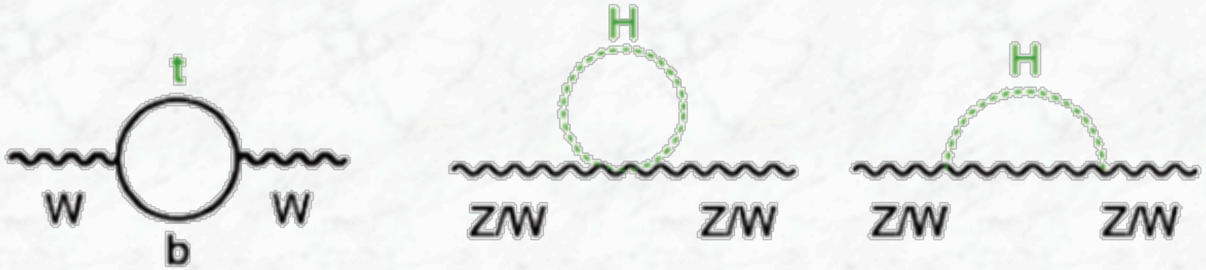
$$A_{\mathrm{FB}} \sim g_{\mathrm{A}}^{e} g_{\mathrm{A}}^{f} \cdot \frac{s(s-M_{\mathrm{Z}}^2)}{(s-M_{\mathrm{Z}}^2)^2+M_{\mathrm{Z}}^2\Gamma_{\mathrm{Z}}^2}$$

- A_{FB} can be used for the determination of the fermion couplings



LO Standard Model prediction:
$g_A = T_3$
$g_V = T_3 - 2\,Q \sin^2\theta_W$

Electroweak radiative corrections



Standard Model relations (lowest order)

$$\rho = \frac{m_{\mathrm{W}}^2}{m_{\mathrm{Z}}^2 \cos^2\theta_{\mathrm{W}}} = 1$$

$$\sin^2\theta_{\mathrm{W}} = 1 - \frac{m_{\mathrm{W}}^2}{m^2\mathrm{Z}}$$

$$m_{\mathrm{W}}^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_{\mathrm{W}} G_{\mathrm{F}}}$$

$$\alpha(0)$$

Relations including radiative corrections

$$\vec{\rho} = 1 + \Delta\rho$$

$$\sin^2\theta_{\mathrm{eff}} = (1 + \Delta\kappa)\sin^2\theta_{\mathrm{W}}$$

$$m_{\mathrm{W}}^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_{\mathrm{W}} G_{\mathrm{F}}} \cdot \frac{1}{(1-\Delta r)}$$

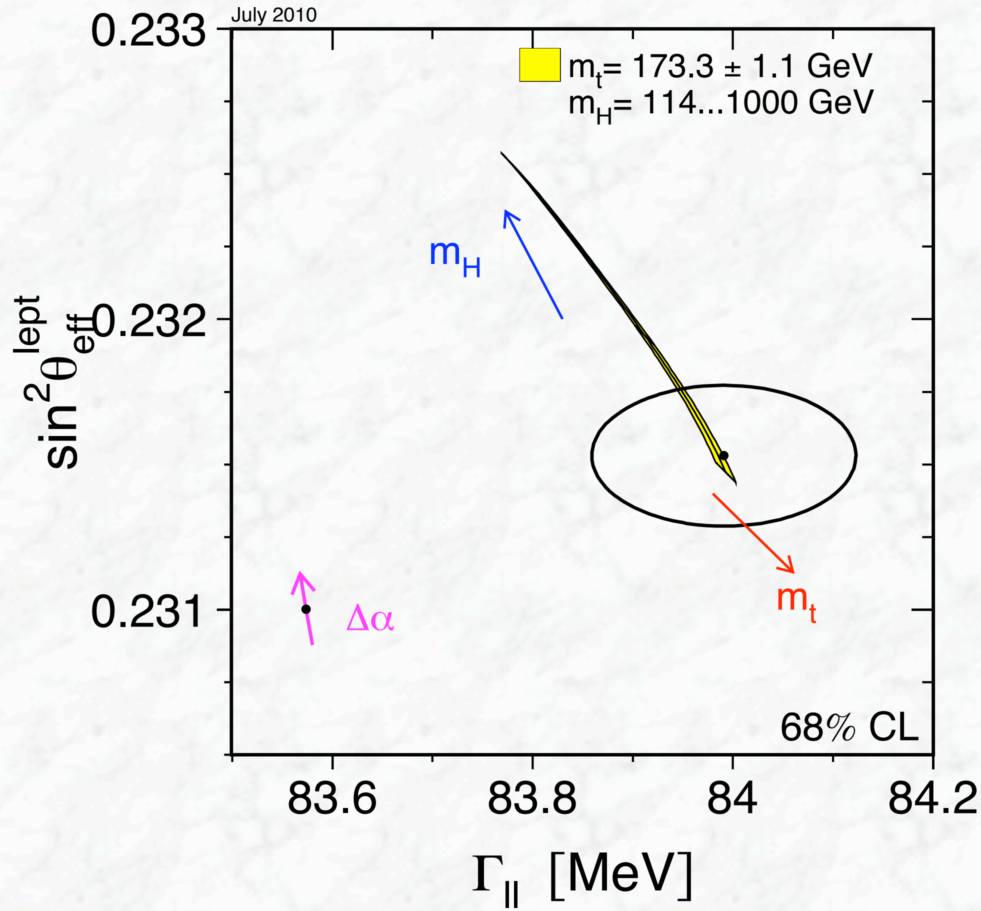
$$\alpha(m_{\mathrm{Z}}^2) = \frac{\alpha(0)}{1-\Delta\alpha}$$

$$\Delta\alpha = \Delta\alpha_{\mathrm{lepl}} + \Delta\alpha_{\mathrm{top}} + \Delta\alpha_{\mathrm{had}}^{(5)}$$

$$\Delta\rho, \Delta\kappa, \Delta r = f(m_t^2, \log(m_{\mathrm{H}}), \ldots)$$

Results of electroweak precision tests at LEP (cont.)

partial decay width versus $\sin^2 \theta_W$:

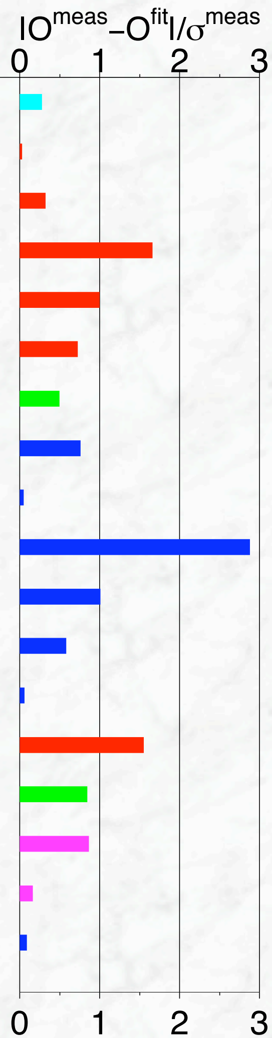


Results of electroweak precision tests at LEP (cont.)

Summary of results:

- All measurements in agreement with the Standard Model
- They can be described with a limited set of parameters

	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02768
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ_{had}^{0} [nb]	41.540 ± 0.037	41.479
R_l	20.767 ± 0.025	20.742
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1038
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.399 ± 0.023	80.379
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.3 ± 1.1	173.4



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