Problem set for the lecture
**Particle Detectors, WS 2015/16**

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# Problem set 3

Deadline: Thursday November 12, 10am
(Please drop into mailbox number 1 on the ground floor of the Gustav-Mie building.)

1. **Compton scattering**
The total cross-section of Compton scattering can be described using the Klein-Nishina formula:

$$\sigma_C(\epsilon) \;=\; 2\pi r_e^2 \left[\frac{1+\epsilon}{\epsilon^2}\;\left(2\frac{1+\epsilon}{1+2\epsilon}\;-\frac{1}{\epsilon}\ln(1+2\epsilon)\right)\;+\;\frac{1}{2\epsilon}\ln(1+2\epsilon)\;-\;\frac{1+3\epsilon}{(1+2\epsilon)^2}\right]$$

where $\epsilon$ is the so-called reduced photon energy $\epsilon \;=\; \frac{E_\gamma}{m_e c^2}$.

Prove that for small photon energies, classical Thomson scattering results from the Klein-Nishina formula:

$$\lim_{\epsilon\to 0}\; \sigma_C(\epsilon) \;=\; \frac{8\pi}{3} r_e^2 \;=\; \sigma_{Thomson}.$$

*Hint: Consider bringing at least two suitable terms to a common denominator and using L'Hôpital's rule.* (3 Points)

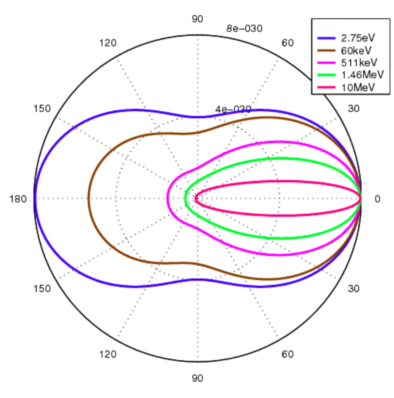


Figure 1: The angular dependence of the Klein-Nishina cross-section for a range of energies.

2. **Cherenkov radiation**

(a) Different types of particles can be identified using Cherenkov radiation if the refractive index of the medium is chosen appropriately. What refractive index has to be chosen to separate kaons from pions at a momentum of 10 GeV/c with a threshold Cherenkov counter?

(b) The number of photons emitted per unit path length in the wavelength range $\lambda_1$ to $\lambda_2$ is found to be

$$\frac{dN}{dx} = 2\pi\alpha z^2 \int\limits_{\lambda_1,\ (n(\lambda)>\frac{1}{\beta})}^{\lambda_2} \left(1 - \frac{1}{\beta^2\ n(\lambda)^2}\right) \frac{d\lambda}{\lambda^2} ,$$

where $\alpha$ is the fine structure constant and $z$ the charge of the particle in units of elementary charge.

Calculate the number of photons emitted in the visible spectrum ($\lambda_1 = 400$ nm, $\lambda_2 = 700$ nm), depending on the Cherenkov angle $\theta_C$ and neglecting dispersion ($n = const.$).

(c) Consider a Cherenkov detector with aerogel as radiator ($n = 1.035$) to separate pions, kaons and protons whose momentum was measured to be 10 GeV/c with a different detector preceding the aerogel. Assume a photon detection efficiency of 30% within the the visible spectrum and 0 outside. For simplicity use a Gaussian distribution for the number of detected photons per particle (with a standard deviation of $\sqrt{N}$). A particle is considered identified if its signal (number of detected photons) is within two standard deviations of its expected mean. What is the minimum length of the radiator to separate the signals for the above particle types with a misidentification rate of less than 2.5%?

(7 Points)