Problem set for the lecture
Particle Detectors, WS 2015/16

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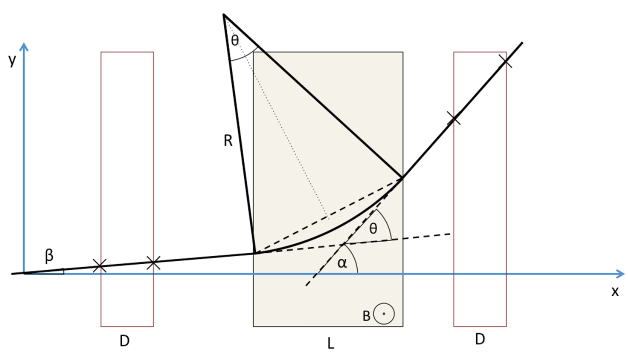
Problem set 7

Deadline: Thursday December 10, 10am
(Please drop into mailbox number 1 on the ground floor of the Gustav-Mie building.)

1. **Dipole spectrometer**



The momentum of relativistic particles is measured using a setup consisting of two tracking detectors of length D each, with a dipole magnet in between. The magnetic field is assumed to be uniform and constrained to a volume with length L. The momentum is calculated from the deflection angle θ between the two straight tracks measured before (having an angle β) and after (α) the magnetic field. Due to a very high p_x of incident particles, all three angles are assumed to be small.

(a) The tracking detectors consist of two tracking layers each, with a distance D between the two layers providing a 'lever arm' to measure the path of the particle. Use the error on the measurement of the slope of the path

$$\sigma_{slope} = \frac{\sigma_x}{D}\sqrt{\frac{12(N-1)}{N(N+1)}}$$

where N is the number of tracking layers, to determine the error on α, β and, using error propagation, θ.

(b) The momentum p can be calculated from $p/(\mathrm{GeV/c}) = 0.3 \cdot (B/\mathrm{T}) \cdot (R/\mathrm{m})$, where R is the radius of the curved path inside the magnetic field. First provide $\frac{dp}{d\theta}$ and then, given the result of (a), the relative error $\frac{\sigma_p}{p}$ on the momentum measurement.

(c) What is the relative error for $\sigma_x = 100\mu\text{m}$, $D = 1$ m, $B = 1.5$ T and $L = 2$ m, for muons with a momentum of either 10 or 100 GeV/c? Usually at least three tracking layers are used to allow precise alignment of the layers. By what factor is the error reduced when an extra tracking layer is inserted in between the other two for both tracking detectors?

(4 Points)

2. **Sign of charge of a particle**
The path of a particle is measured at N equidistant tracking layers inside a solenoid field.

(a) Derive the maximum transverse momentum p_T for which the sign of the charge of an electron can be determined with a significance of two sigma, given

$$\sigma_\kappa \approx \frac{\sigma_x}{L^2}\sqrt{\frac{720}{N+4}} \quad , \kappa = \frac{1}{R}$$

(b) What number results for $N = 12$, $L = 1.15$ m, $B = 2$ T, $\sigma_x = 20\mu\text{m}$?

(2 Points)

3. **NUMERICAL SIMULATION - Diffusion in a Time Projection Chamber**
A time projection chamber consists of two cylindrical drift volumes with a length of 1 m each, which are filled with argon gas at room temperature. The electric and magnetic field both point along the z-direction of the cylinder. A simplified ("toy") model of the diffusion of localized charges over the long drift lengths is to be implemented, to study the impact of the magnetic field.

- Assume the distance between each interaction of an electron with gas atoms to follow an exponential decay distribution with a mean free length $\lambda_e = 2.7\mu\text{m}$. It is sufficient to simulate diffusion in the x-y plane only. For simplification assume that all electrons always have, in x-y, exactly the mean thermal speed. Each collision with a gas atom scatters an electron into a random direction.
- In contrast to problem set 6, use known relations for the motion of the electrons, e.g. having the algorithm directly step from one collision to the next and calculating the change in position from the curvature and length of the path/arc.
- Simulate electrons from ionization of the gas at the central electrode disk, with a drift speed of electrons of $v_z = 5$ cm/μs. Study the root-mean-square deviation of the electron points of impact on the endplates for B-field strengths between 0 and 3 T and electrons originating in the same point. What is the relation between the RMS and the strength of the magnetic field? Does the simple model adequately describe the behaviour specified in the lecture?

(4 Points)