Problem set for the lecture

Particle Detectors, WS 2015/16

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Problem set 1

Deadline: to be announced during the first lecture

(Please drop into mailbox number 1 on the ground floor of the Gustav-Mie building.)

1. **Accelerator kinematics**
   The LHC accelerator collides two beams of protons, each with an energy of 8 TeV.
   (a) What is the centre-of-mass energy of the collisions?
   (b) If one wanted to achieve the same centre-of-mass energy by shooting one proton beam at a fixed target, what beam energy would be required?

   (2 Points)

2. **Relativistic decay kinematics**
   Consider neutral particles moving at relativistic speeds in the laboratory system, decaying in flight into two photons. Due to momentum conservation, in the rest frame of one of the neutral particles the two decay products are back-to-back (at an angle of 180° to each other).
   (a) For one such decay, the two photons are observed to have the energies $E_1 =$ 100 GeV and $E_2 =$ 156 GeV and an angle $\alpha = 60°$ between them. What was the energy E_0 and the rest mass m_0 of the initial particle?
   (b) For an initial particle with the E_0 and m_0 resulting above, what is, in the laboratory system, the minimum angle α_{min} between any two decay photons that is kinematically possible?
   Hint: What relation between E_1 and E_2 results for the minimum angle?
   (c) Again for a particle with the above E_0 and m_0, what is the Lorentz factor γ and ratio of particle velocity to the speed of light β?
   (d) For the above E_0 and m_0, what are the minimum and maximum energies of decay photons that can be observed in the laboratory system, and what is the corresponding angle θ of the photons to the z-axis in the rest frame of the di-photon system, assuming the decayed particle to have moved along the z-axis?

   (4 Points)

3. **Dating methologies**

The age of minerals can be determined from the ratio of parent nuclei to daughter isotopes resulting from a radioactive decay with a decay constant λ_1. For heavy nuclides, the daughter isotopes decay further with a decay constant λ_2, resulting in a so-called decay chain.

(a) Compile the differential equations for the change (as a function of time) of number of parent nuclei $N_1(t)$ and daughter nuclei $N_2(t)$, and solve them with the ansatz:

$$N_1(t) = C_{11}e^{-\lambda_1 t}$$

$$N_2(t) = C_{21}e^{-\lambda_1 t} + C_{22}e^{-\lambda_2 t}$$

for the initial condition of $N_2(t = 0) = 0$.

(b) Explain the ^{14}C method of dating organic samples.

(4 Points)