## Übungen zu Physik an Hadron-Collider SS 2011 <br> Prof. Karl Jakobs, Dr. Iacopo Vivarelli <br> Übungsblatt Nr. 3

Die Lösungen müssen bis 11 Uhr am Donnerstag den 26.5.2011 in die Briefkästen im Erdgeschoss des Gustav-Mie-Hauses eingeworfen werden!

## 1. Kinematic variables - 1

At a hadron collider, if a massive particle decays into a lepton and a neutrino, its invariant mass cannot be reconstructed, as the longitudinal component of the neutrino momentum cannot be measured.

- How is the transverse momentum of the neutrino measured? [ $\mathbf{1}$ point]

A useful variable to consider is the transverse mass $M_{T}$, defined as:

$$
\begin{equation*}
M_{T}{ }^{2}=\left(E_{T}(1)+E_{T}(2)\right)^{2}-\left(\mathbf{p}_{T}(1)+\mathbf{p}_{T}(2)\right)^{2} \tag{1}
\end{equation*}
$$

- Derive a simplified formula for the transverse mass in the approximation $m_{1}=m_{2}=0$ [1 point]

We now consider a $W$ boson with mass $M_{W}=80 \mathrm{GeV}$ and its decay $W \rightarrow e \nu$ (there is no need here to distinguish the $W^{+} \rightarrow e^{+} \nu$ and the $W^{-} \rightarrow e^{-} \bar{\nu}$ ). Assume that the $W$ is produced at rest.

- Determine the differential distribution $d N / d M_{T}$ and its dependency on $M_{T}$. Show that the distribution has an end point at $M_{T}=M_{W}$ [ $\mathbf{3}$ points] [HINT: the following identity

$$
\begin{equation*}
\frac{d N}{d M_{T}}=\frac{d N}{d \Omega} \frac{d \Omega}{d M_{T}} \tag{2}
\end{equation*}
$$

can be useful.]

## 2. Kinematic variables - 2

- Show that the pseudorapidity $\eta=-\ln \tan (\theta / 2)$ is a good approximation for the rapidity $y=\tanh ^{-1}\left(p_{z} / E\right)$ if the particle mass is much smaller than its momentum ( $\theta$ is the polar angle with respect to the beam line). [2 points]
- Write down explicitly the equations to transform from a $(x, y, z)$ coordinate system to a ( $r_{T}, \eta, \phi$ ) one ( $r_{T}$ being the projection on the transverse $x y$ plane of the spherical coordinate $r$ ). [2 point]


## 3. Two particle kinematics

Let's consider a proton proton collision. The reference frame we consider (lab frame) is the proton-proton CM, in which each proton has momentum $|\mathbf{p}| \gg m_{p}$ ( $m_{p}$ being the mass of the proton). The two colliding partons carry a fraction $x_{1}, x_{2}$ of the initial proton momentum. Assume the two partons are massless.

- Compute the invariant mass $M$ of the parton-parton system in terms of $P, x_{1}, x_{2}[\mathbf{1}$ point]

Let's assume that an object with mass $M$ is indeed produced, and it decays into massless particles.

- Compute the differential angular distribution $d N /(d \phi d \eta)$ in the center of mass frame of the produced particle ( $\eta$ being the pseudorapity computed with respect to the beam axis). [3 points]
- What is the distribution in the lab frame? [ $\mathbf{1}$ point]

